Karlsruhe Institute of Technology

High Performance Humanoid Technologies

Robotics I: Introduction to Robotics
Chapter 5 – Control of Robot Systems

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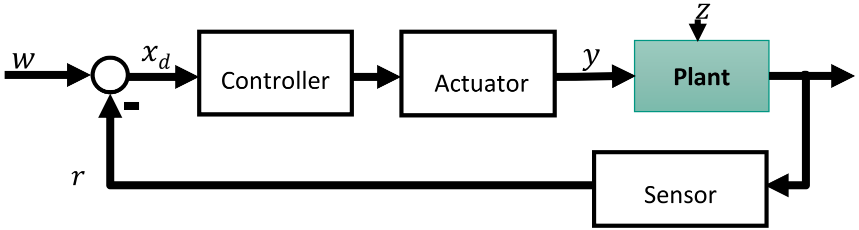
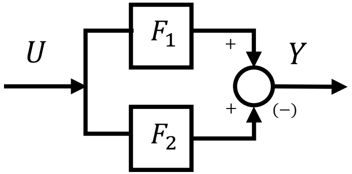
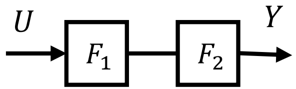
https://www.humanoids.kit.edu







$$L\{f(t)\} = F(s) = \int_0^{\infty} f(t)e^{-st}dt$$

Contents

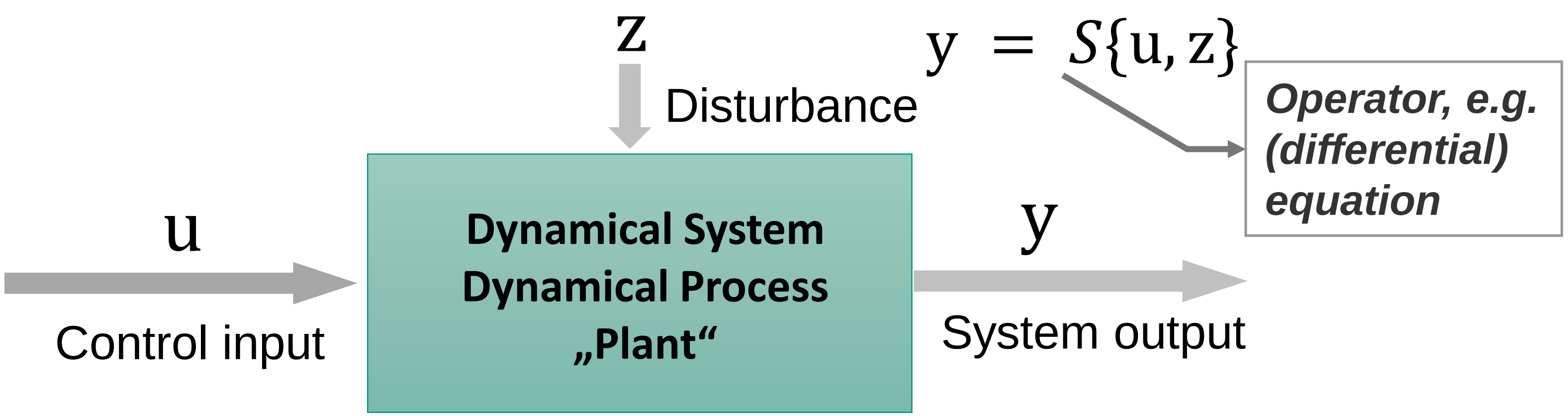
Control Theory

- **Control theory (Regelungstechnik):**
  Theory of automatic, goal-orientated influencing of dynamic, time-dependent processes at run-time

- **Fundamental situation in control theory:**
  Design of a system for automatic, targeted influencing a process with incomplete system knowledge, in particular in the presence of disturbances

- Methods of control theory are universally applicable, independent of the specific nature of the given system

Structure and Operation of a Control System



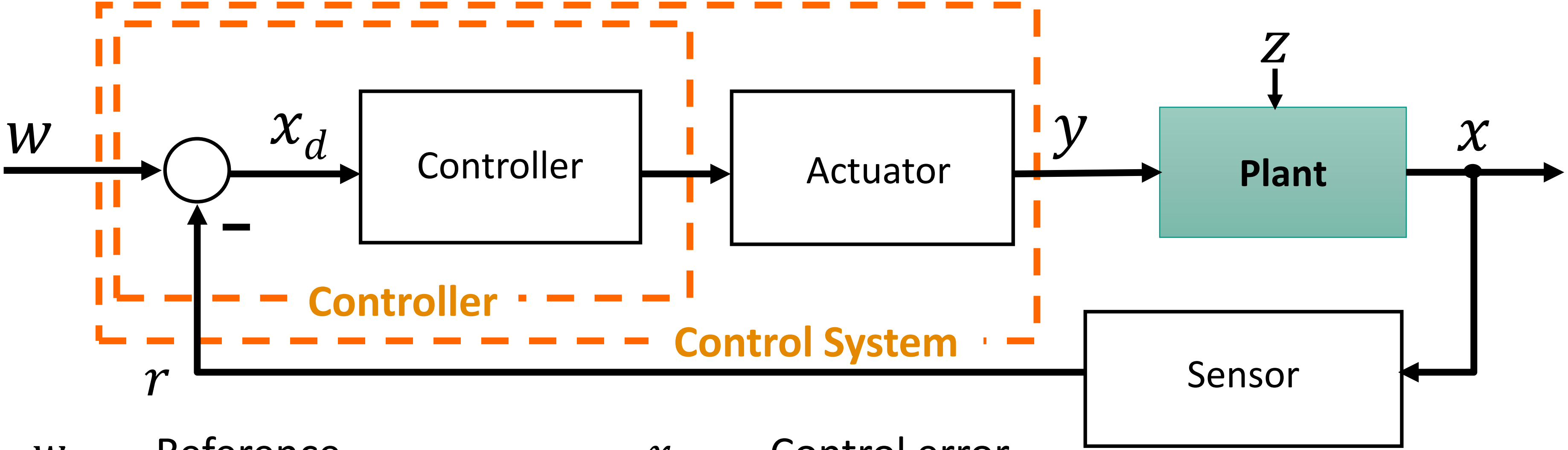
- **Task:**
  The system output is to be influenced via the control input in such a way that a desired system behavior (i.e. system output) is achieved, despite a disturbance that is not or only partially known

Structure and Operation of a Control System

- **Principle of operation:**
  The plant is to be observed continuously, and the obtained information is used to change the system input variable in such a way that its output variables matches the desired output as close as possible, despite the effects of the disturbance.

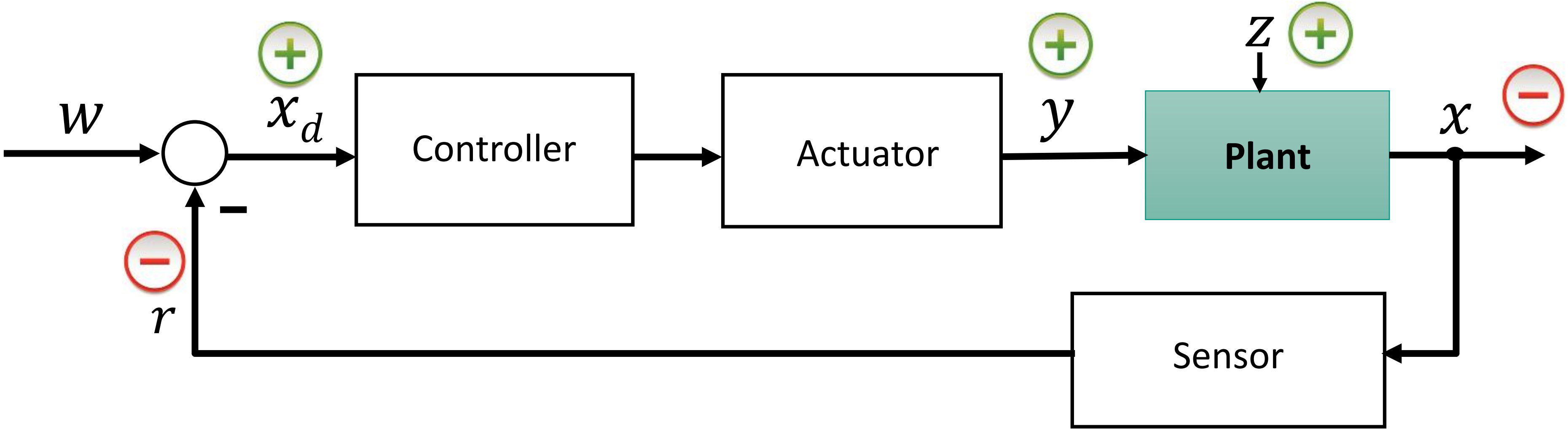
  A system that can achieve this is called a **closed-loop control system (Regelung)**.

Structure of a Control System



w	Reference	x_d	Control error
y	Control Input	x	System output
r	Feedback	z	Disturbance

Structure of a Control System



Target value of x: x_s

Measurement: $r = K_j\, x \qquad K_j > 0$ (constant)

Selection of reference: $w = K_j\, x_s$

Then: $x_d = w - r = K_j\, x_s - K_j\, x = K_j\, (x_s - x)$

Operation of a Control System

Desired value of x: x_s

Reference value: $r = K_j\, x$ $\quad K_j > 0$ (constant)

Measured value of x: $w = K_j\, x_s$

Then: $x_d = w - r = K_j\, x_s - K_j\, x = K_j\,(x_s - x)$

Initial Situation: $x = x_s \Rightarrow x_d = 0$ (stationary system)

z becomes larger $\Rightarrow$ x decreases $\Rightarrow$

r decreases $\Rightarrow$ x_d increases $\Rightarrow$

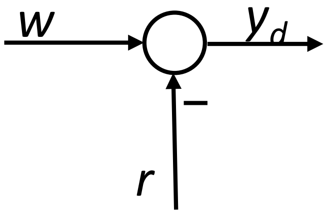
y increases $\Rightarrow$ x increases to resemble the desired value x_s

In short: **The disturbance is regulated.**

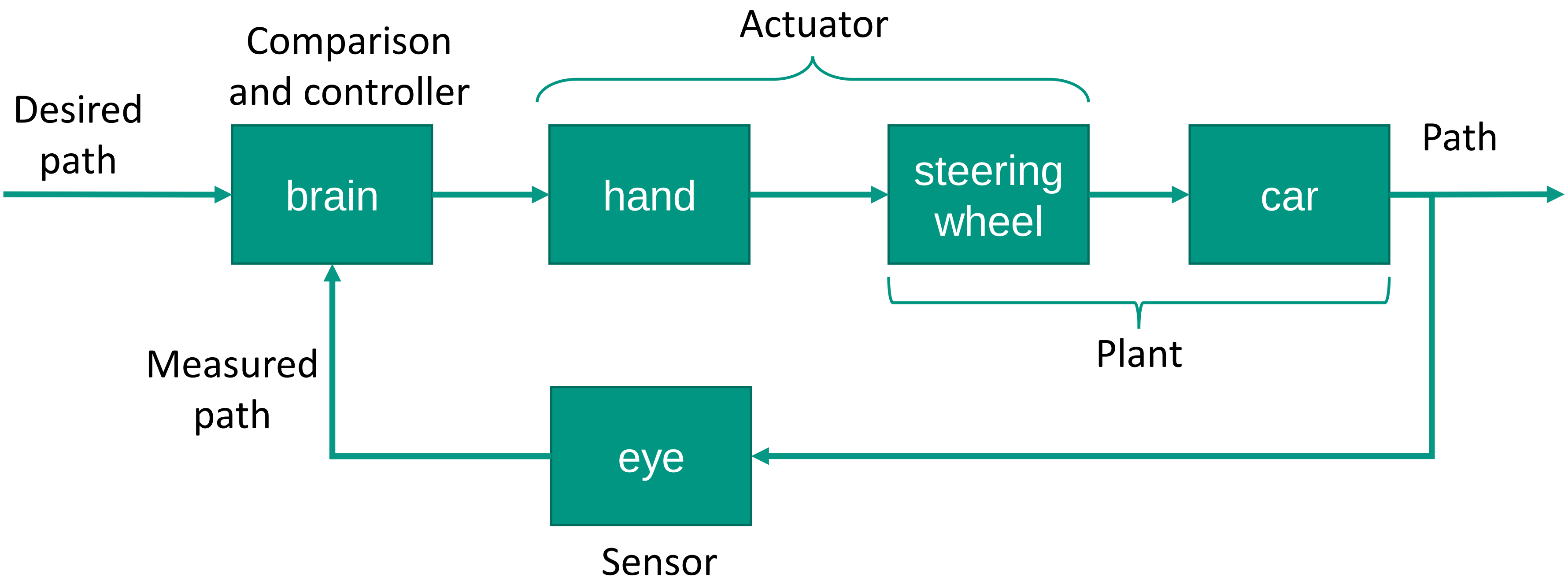
Operation of a Control System

- The system output is changed to resemble the reference value, i.e. the output follows the reference value.
- The control system is a closed loop: **Closed loop control**
- Essential:

Feedback is SUBTRACTED from the reference



Example: Car Steering as Control System



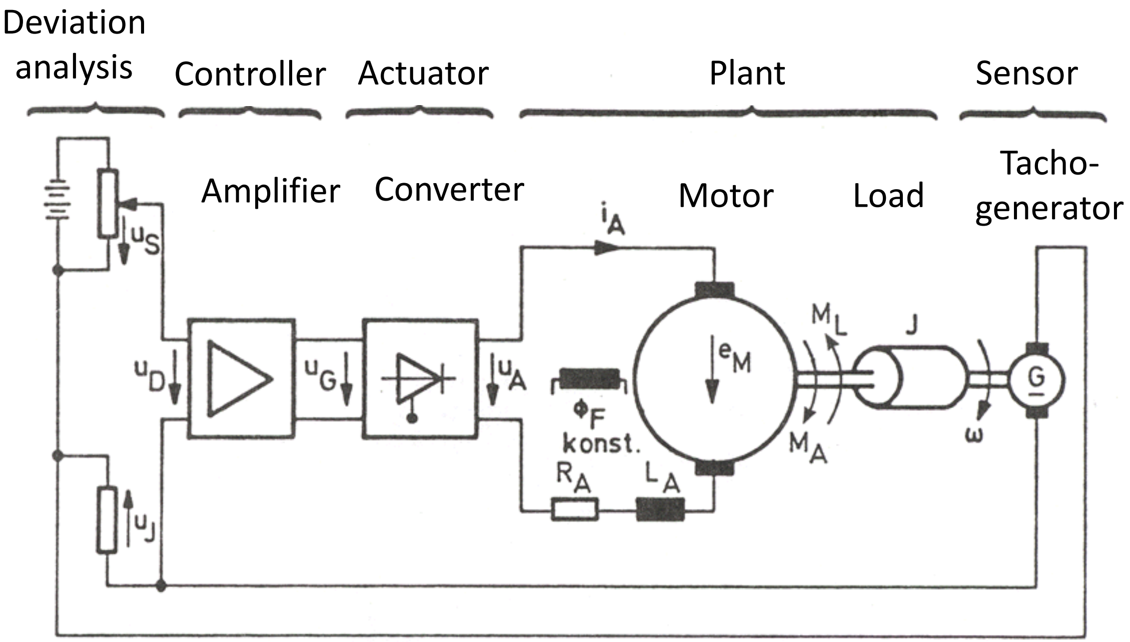
*German original taken from: **Regelungstechnik; O. Föllinger***

Definition: Control System

A control system is an arrangement that continuously observes the plant‘s output, computes the deviation from a reference value and uses this error to adjust the system output to match the reference.

This is achieved with only **incomplete** knowledge about the plant and, especially, about the disturbance.

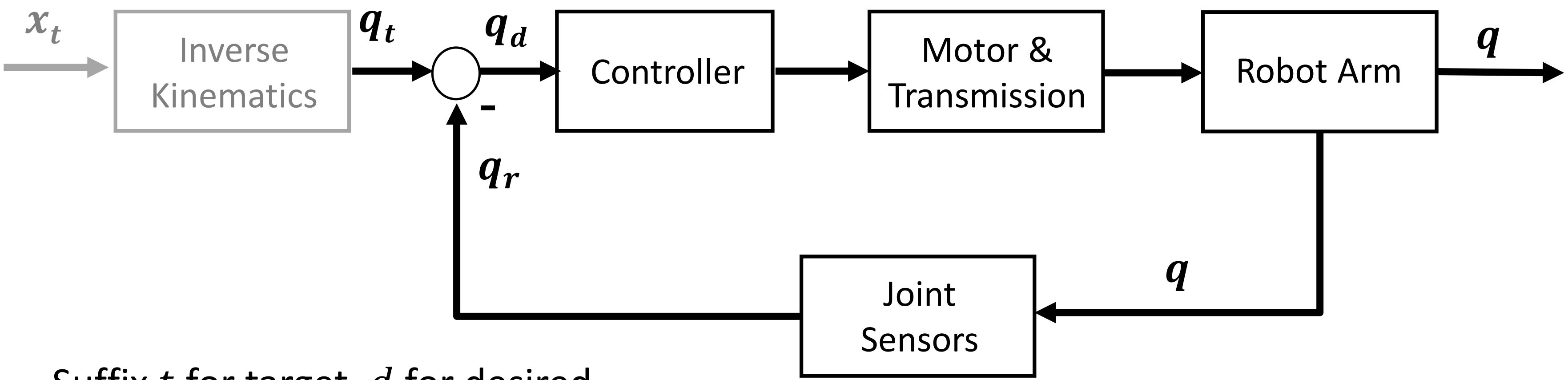
Example: Speed control of a DC motor



German original taken from: **Regelungstechnik; O. Föllinger**

Example: Control in the Joint Angle Space

- Control variables for the joint actuators are generated from the target and measured joint angles



Suffix t for target, d for desired

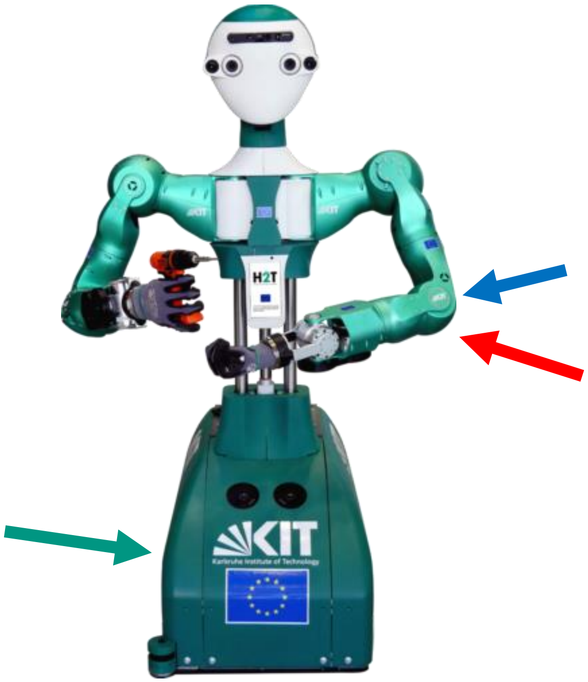
$\boldsymbol{q}$: joint angles

$\boldsymbol{x}$: target in Cartesian space

Example: ARMAR-6

- High Level: **Computer**
  - Central control of the joints
  - Position (e.g., from inverse kinematics)
  - Velocity (e.g., from inverse kinematics)
  - Torque (e.g., from inverse dynamics)
  - **EtherCAT-Bus** (1000 Hz)

- Low Level: **Motor Controllers**
  - Control (up to 20 kHz) for
    - **PWM**
    - **current**

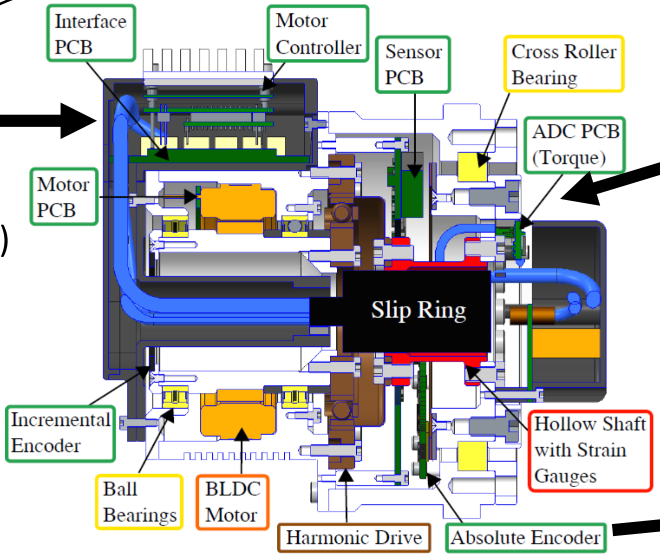


- Measurement of position, torque and current in the **joint**

Example: ARMAR-6

y

Setpoint
(from computer)



z

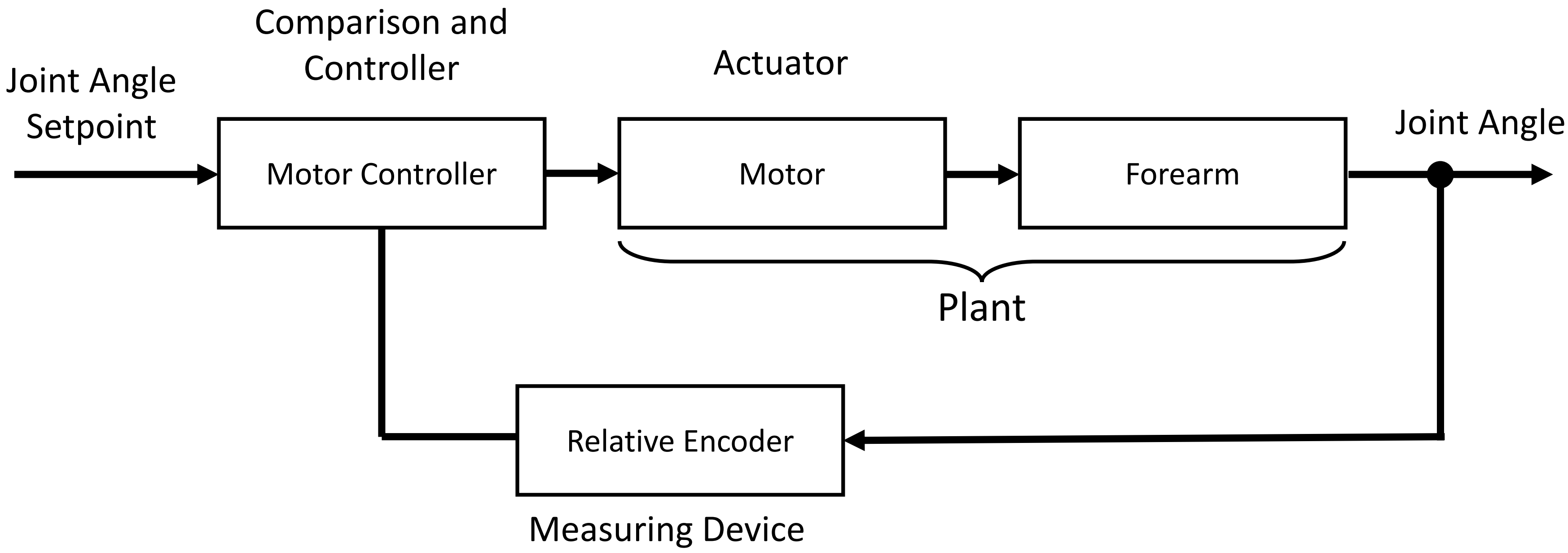
Disturbance variable
(e.g. external forces)

Output variable
(joint angle)

x

Dynamical System

Example: ARMAR-6

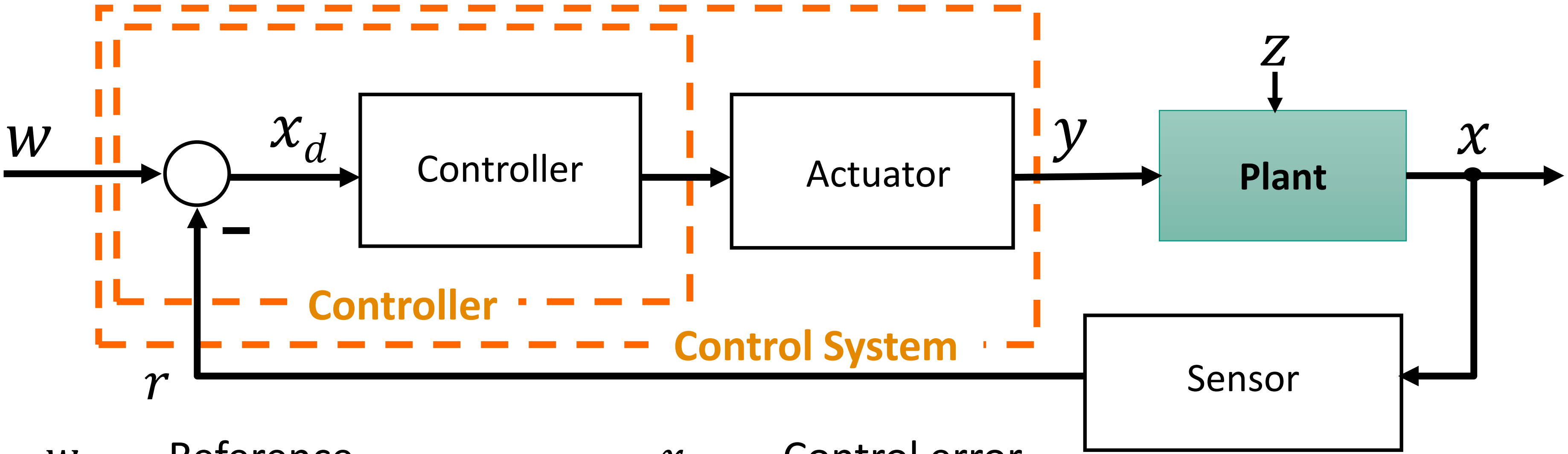


Control Loop

Block diagram of a control system:

- From physical laws, we can derive **equations (differential or difference equations) that describe the relationships between time-varying quantities of the system**.
- The time-varying quantities and their equations are represented by suitable symbols.
- A block in the block diagram uniquely assigns each time response of the input variable to a time respones of the output variable, thus acting as a transfer element.

Structure of a Control System



w	Reference	x_d	Control error
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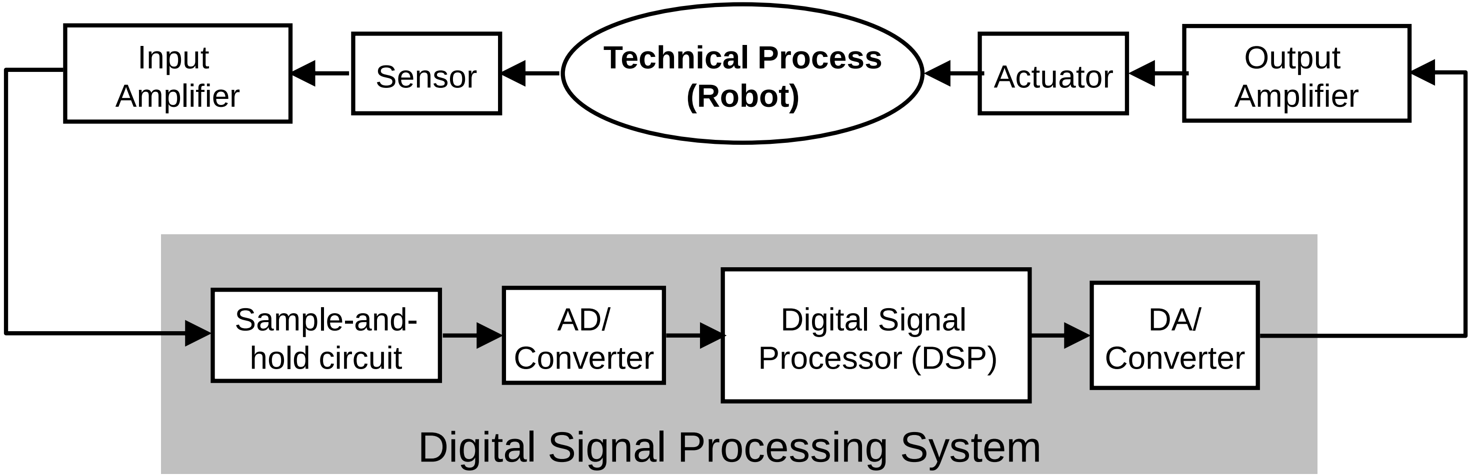
Contents

Diagram of Digital Signal Processing Systems



- Information acquisition using sensors
- “Digitize” sensor data
- Algorithms of digital signal processing
- Convert processed signal back into an analog signal

Diagram of Digital Signal Processing Systems

- **Input amplifier** to amplify the sensor signal and convert it to the required voltage range

- **Sample-and-hold** element for the periodic sampling of the input signal. The sampled value is held constant within a sampling period

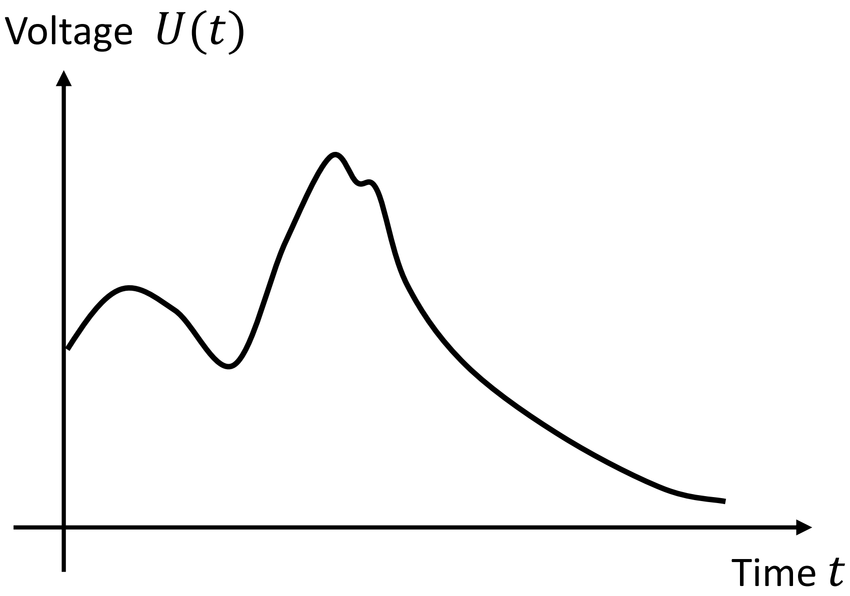
- Input amplifier with *anti-aliasing filter* to eliminate high interference frequencies from the sensor signal

- The **output amplifier** smooths the signal from the DA converter (reconstruction filter)

Continuous and Discrete Signals (1)

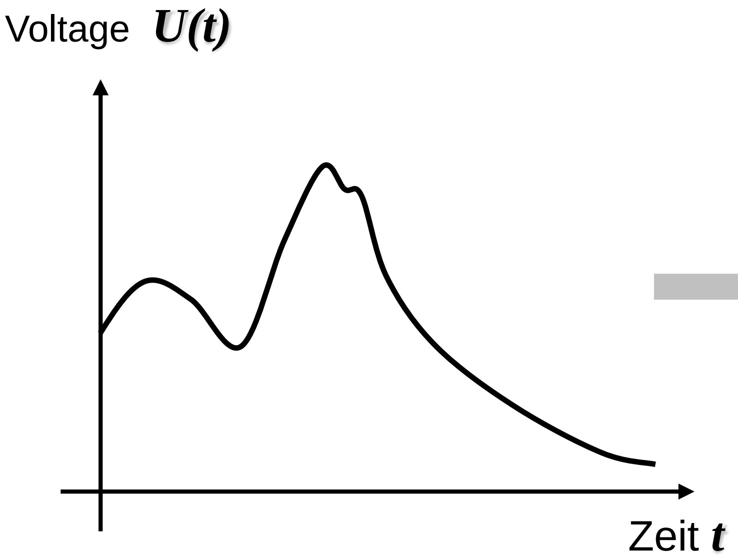
Signal as a physical carrier of information

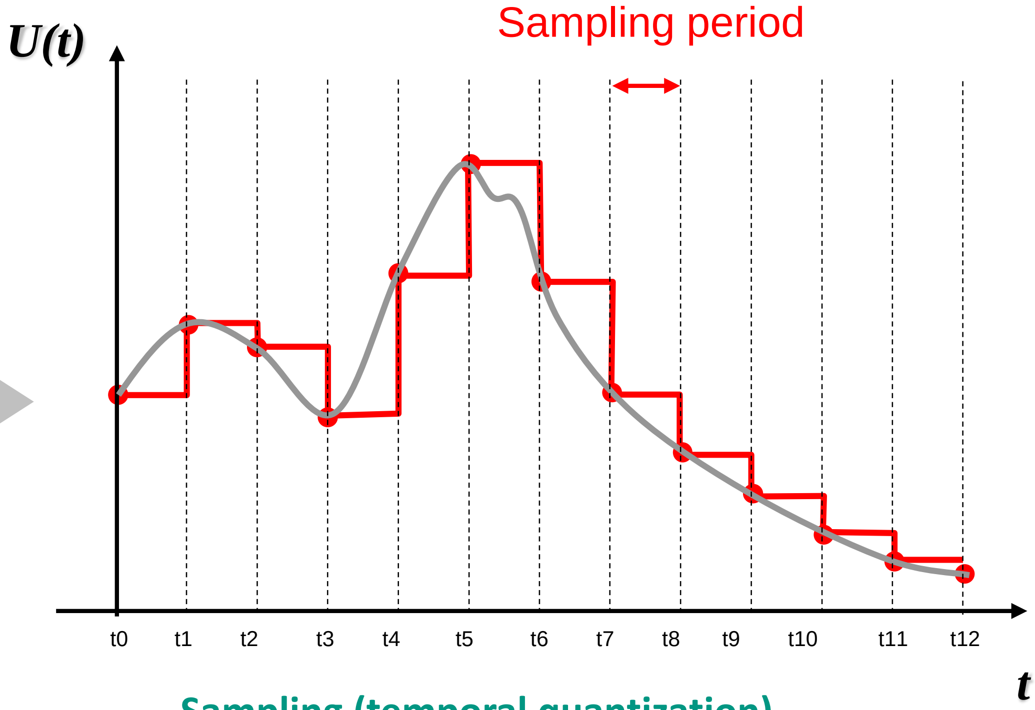
- A signal is a function of an independent variable t, which usually represents time. The signal is represented as $U(t)$.
- Analog Signal: $U(t)$ is defined at every moment and can take any arbitrary value (signal with continuous values).



Analog Signal
Continuous in time and value

Continuous and Discrete Signals (2)





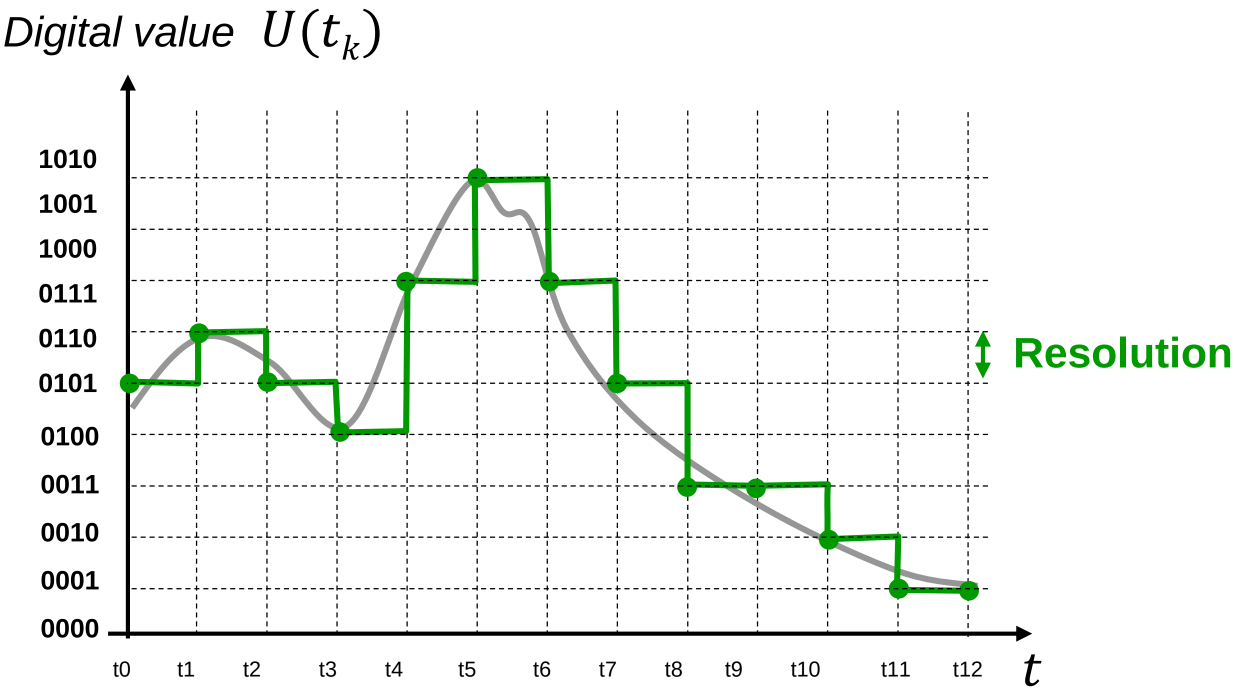
Analog Signal
Continuous in time and value

Sampling (temporal quantization)
Discrete time and continuous value

Continuous and Discrete Signals (3)

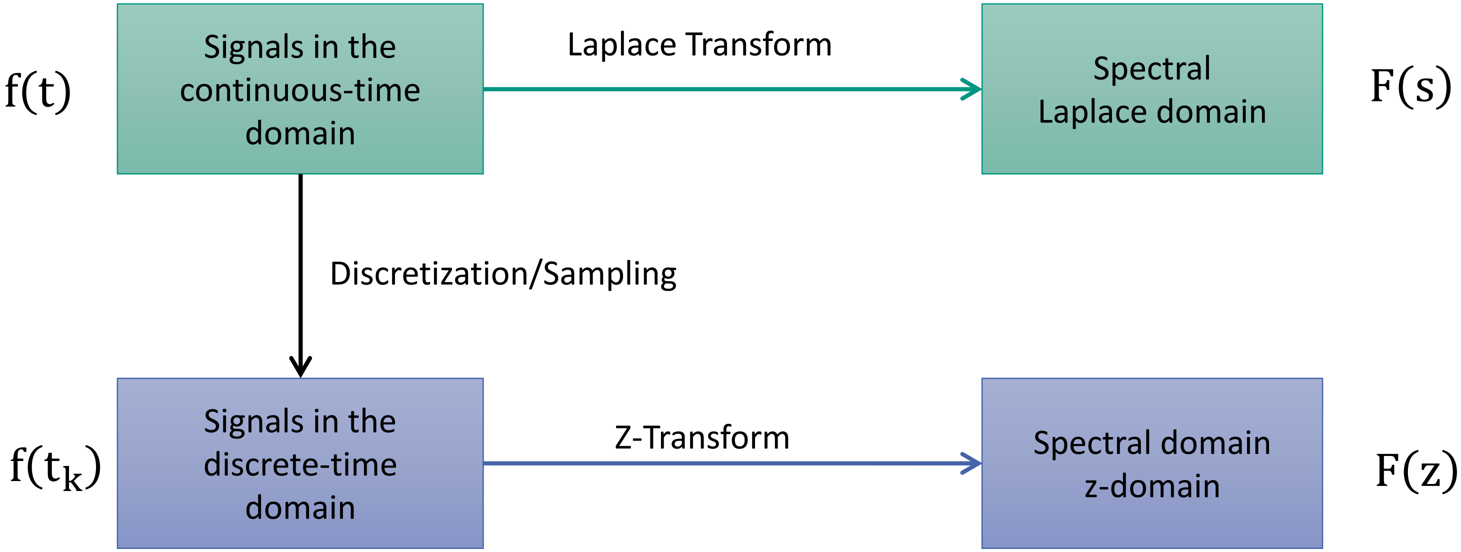
- Signal $U(t_k)$ with a finite number of different values

- Important: Signals with two different values



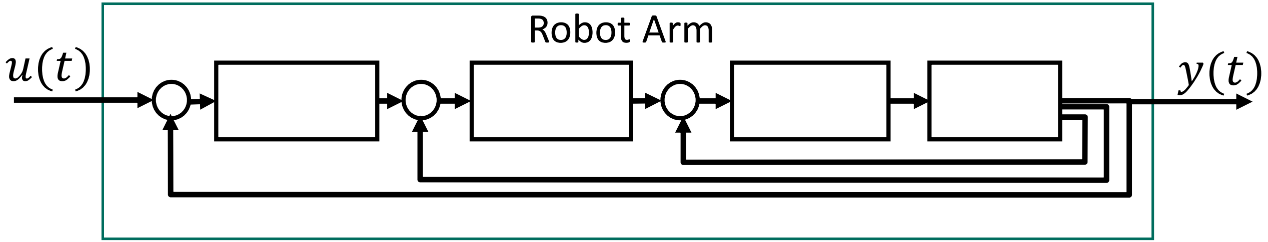
Amplitude quantization
Discrete in time and value

Description of Dynamic Systems



Fundamentals of Control

- **Example:** Position control for a robot joint



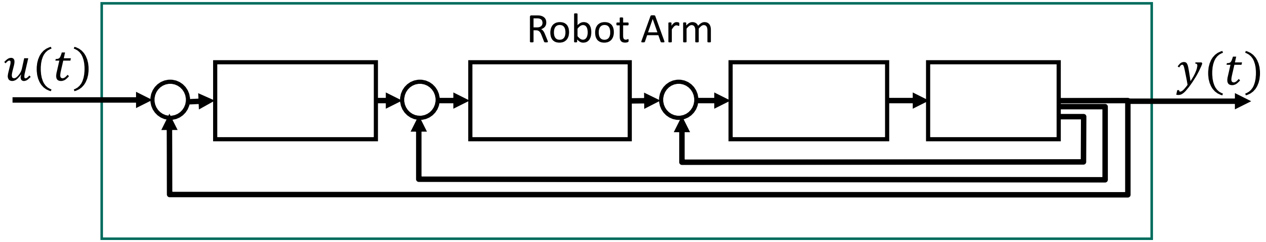
  - Input signal $u(t)$: Desired position (reference variable, setpoint)
  - Output signal $y(t)$: Actual position (process variable)

- **Objective:** Describe output signals for a given input signal

- **Procedure:**
  1. Description of the system with differential equations (or difference equations)
  2. Transform into the frequency domain (Laplace)
  3. Deriving the transfer function

Fundamentals of Control

- Example: Position control for a robot joint



- Input signal $u(t)$: Desired position (reference variable, setpoint)
- Output signal $y(t)$: Actual position (process variable)

Transfer Function: Application

- Transform to the frequency domain (**Laplace Transform**)

$$L[u(t)] = U(s) \qquad\qquad L[y(t)] = Y(s)$$

- **Transfer Function**

$$\mathrm{G(s)} = \frac{Y(s)}{U(s)} = \frac{\text{Output}}{\text{Input}}$$

- The transfer function is important for the controller design:
  - Analysis of the system behavior with different input signals
    → Example: Stability analysis
  - Determination of the controller parameters
    → Optimization of the parameters for the given system

Contents

Laplace Transform

$$\boldsymbol{L\{f(t)\} = F(s) = \int_0^{\infty} f(t)e^{-st}dt} \qquad s := \sigma + j\omega; \quad f(t) = 0, t < 0$$

- Differential and integral expressions are replaced by **algebraic expressions**
- Solving equation in the **frequency domain** instead of the time domain
- Integral must converge – fulfilled for linear $f(t)$

Laplace Transform

- $f(t) = a$

$$\mathcal{L}[f(t)] = \mathrm{F}(s) = \int_0^{\infty} f(t) \cdot e^{-st} dt$$

Laplace Transform

- $f(t) = a$

$$\mathcal{L}[f(t)] = \mathrm{F}(s) = \int_0^{\infty} f(t) \cdot e^{-st} dt$$

$$\mathcal{L}[a] = \int_0^{\infty} a \cdot e^{-st} dt = a \cdot \int_0^{\infty} e^{-st} dt$$

$$\mathcal{L}[a] = a \cdot \left[-\frac{1}{s} \cdot e^{-st}\right]_0^{\infty} = a \cdot \left(0 - \left(-\frac{1}{s} \cdot 1\right)\right)$$

$$\mathcal{L}[a] = \frac{a}{s}$$

Laplace Transformation of $f(t) = t$

- $f(t) = t$

$$\mathcal{L}[f(t)] = \mathrm{F}(s) = \int_0^\infty f(t) \cdot e^{-st} dt$$

$$\mathcal{L}[t] = \int_0^\infty t \cdot e^{-st} dt$$

$$\int_0^\infty u(t) \cdot v'(t)\, dt = u(t) \cdot v(t)|_0^\infty - \int_0^\infty u'(t) \cdot v(t) dt$$

← Integration by parts

Laplace Transformation of $f(t) = t$

- $f(t) = t$

$$\mathcal{L}[f(t)] = \mathrm{F}(s) = \int_0^\infty f(t) \cdot e^{-st} dt$$

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$$\int_0^\infty u(t) \cdot v'(t)\, dt = u(t) \cdot v(t)|_0^\infty - \int_0^\infty u'(t) \cdot v(t) dt$$

← Integration by parts

$$u(t) = t,\ u'(t) = 1$$

$$v'(t) = e^{-st},\ v(t) = -\frac{1}{s} \cdot e^{-st}$$

Laplace Transformation of $f(t) = t$

- $f(t) = t$

$$\mathcal{L}[f(t)] = \mathrm{F}(s) = \int_0^\infty f(t) \cdot e^{-st} dt$$

$$\mathcal{L}[t] = \int_0^\infty t \cdot e^{-st} dt$$

$$\boldsymbol{\int_0^\infty u(t) \cdot v'(t)\, dt = u(t) \cdot v(t)|_0^\infty - \int_0^\infty u'(t) \cdot v(t) dt}$$

$$\mathcal{L}[t] = \left[t \cdot \left(-\frac{1}{s} \cdot e^{-st}\right)\right]_0^\infty - \int_0^\infty 1 \cdot \left(-\frac{1}{s} \cdot e^{-st}\right) dt$$

$$u(t) = t,\ u'(t) = 1$$
$$v'^{(t)} = e^{-st}, v(t) = -\frac{1}{s} \cdot e^{-st}$$

$$\mathcal{L}[t] = (0\ - 0) - \left[\frac{1}{s^2} \cdot e^{-st}\right]_0^\infty = 0 - \left(0 - \frac{1}{s^2}\right) = \frac{1}{s^2}$$

Laplace Transformation of $\dot{f}(t)$

- Laplace transform of the time derivative $\dot{f}(t)$

$$\int_0^\infty \boldsymbol{u(t)\cdot v'(t)\,dt = u(t)\cdot v(t)\Big|_0^\infty - \int_0^\infty u'(t)\cdot v(t)dt}$$

$$\mathcal{L}[\dot{f}(t)] = \int_0^\infty e^{-st}\frac{df}{dt}dt =$$

Laplace Transformation of $\dot{f}(t)$

- Laplace transform of the time derivative $\dot{f}(t)$

$$\int_0^\infty \boldsymbol{u(t) \cdot v'(t)\, dt = u(t) \cdot v(t)\Big|_0^\infty - \int_0^\infty u'(t) \cdot v(t) dt}$$

$$\mathcal{L}[\dot{f}(t)] = \int_0^\infty e^{-st} \frac{df}{dt} dt = e^{-st} f(t)|_0^\infty - \int_0^\infty -s \cdot e^{-st} f(t) dt$$

Assumption: $\lim_{t \to \infty} e^{-st} f(t) \to 0$

$$\mathcal{L}[\dot{f}(t)] = s \int_0^\infty e^{-st} f(t) dt - f(0) = s \cdot F(s) - f(0)$$

Laplace Transformation of $\int_0^t f(t)\,dt$

- Laplace transform of $\int_0^t f(t)\,dt$

$$\int_0^\infty \boldsymbol{u(t)\cdot v'(t)\,dt = u(t)\cdot v(t)}\Big|_0^\infty - \int_0^\infty \boldsymbol{u'(t)\cdot v(t)dt}$$

$$\mathcal{L}\left[\int_0^t f(t)\,dt\right]$$

$$= \int_0^\infty \int_0^t f(t)\,dt \cdot e^{-st}\,dt = \int_0^t f(\tau)\,d\tau \cdot (-\frac{1}{s}\cdot e^{-st})|_0^\infty$$

$$-\int_0^\infty \left(-\frac{1}{s}\right)\cdot e^{-st} f(t)dt = \frac{1}{s}\int_0^\infty f(t)\cdot e^{-st}\,dt = \frac{1}{s}F(s)$$

$$\mathcal{L}\left[\int_0^t f(t)\,dt\right] = \frac{1}{s}F(s)$$

Laplace Transform

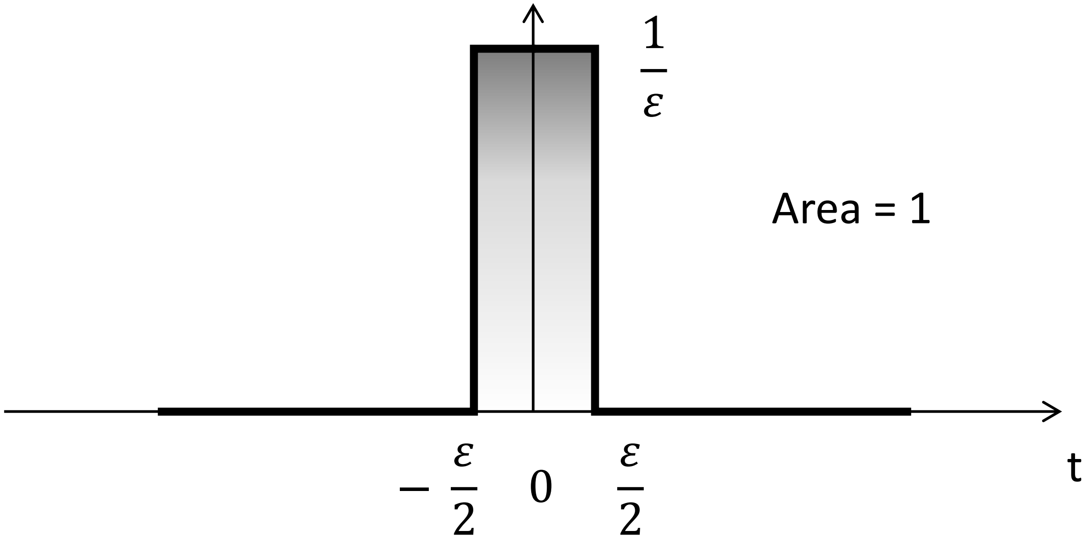
- Laplace Transform of f(t) = $e^{-\alpha t}$

$$\mathcal{L}[e^{-\alpha t}] = \int_0^\infty e^{-\alpha t} \cdot e^{-st}\, dt = \int_0^\infty e^{-(s+\alpha)t}\, dt = \frac{1}{s+\alpha}$$

$$\mathcal{L}[e^{-\alpha t}] = \frac{1}{s+\alpha}$$

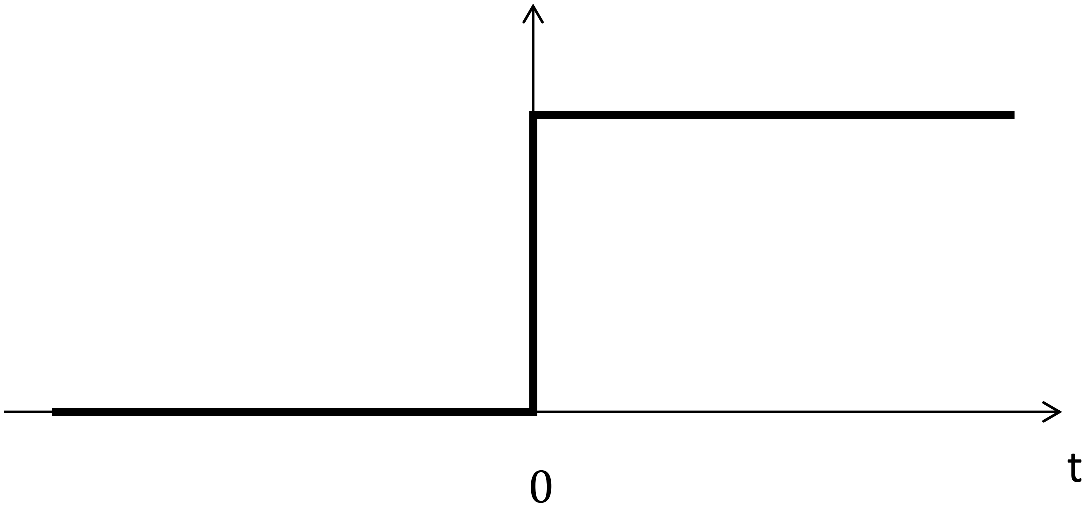
Unit Impulse Function (Dirac Delta Function)

$$\partial(t) = \begin{cases} \infty & , \text{if } \ t = 0 \\ 0 & , \text{if } \ t \neq 0 \end{cases}$$



Unit Step Function

$$\sigma(t) = \begin{cases} 0 & , \text{if } t < 0 \\ 1 & , \text{if } t \geq 0 \end{cases}$$



Laplace Transformation of $\delta(t)$ and $\sigma(t)$

- Laplace transform of $\delta(t)$

$$\mathcal{L}[\delta(t)] = \int_0^\infty \delta(t) \cdot e^{-st} dt = 1$$

- Laplace transform of $\sigma(t)$

$$\mathcal{L}[\sigma(t)] = \int_0^\infty \sigma(t) \cdot e^{-st}\, dt = \int_0^\infty 1 \cdot e^{-st}\, dt = -\frac{1}{s} \cdot e^{-st}\Big|_0^\infty$$

$$s = \delta + j\omega \Rightarrow e^{-st} = e^{-(\delta + j\omega)t} = e^{-\delta t} \cdot e^{-j\omega t}$$
$$= e^{-\delta t} \cdot \underbrace{(\cos \omega t - j \sin \omega t)}_{}$$

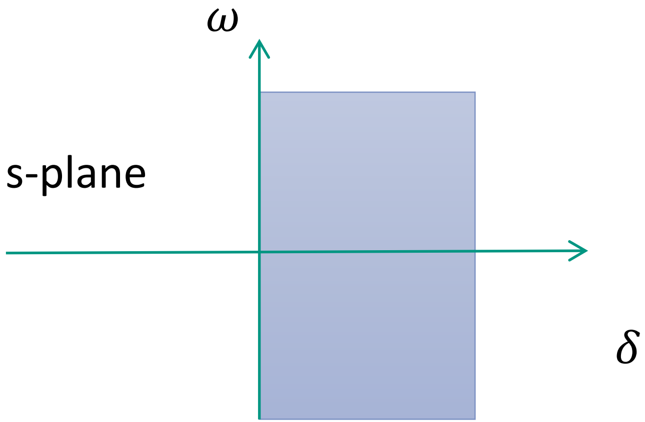
Complex representation of a periodic oscillation

Laplace Transformation

$$e^{-\delta t} = \begin{cases} 0 & for\ \delta > 0 \Rightarrow Oscillation\ diminishes \\ 1 & for\ \delta = 0 \Rightarrow Oscillation \\ -\infty & for\ \delta < 0 \Rightarrow Oscillation\ \ increases \end{cases}$$

Laplace transform of $\sigma\ (t)$ exists only for $\delta > 0$ or $Re(s) > 0$ (right half of the complex plane)

$$\mathcal{L}[\sigma(t)] = \frac{1}{s}$$



Laplace Transform: Properties and Rules

- Linearity $L\{\alpha f_1(t) + \beta f_2(t)\} = \alpha F_1(s) + \beta F_2(s)$
- Convolution: $L\{f_1(t) * f_2(t)\} = F_1(s) \cdot F_2(s)$
- Limit value: $f(t=0) = \lim_{s \to \infty} sF(s)$
- Differentiation: $L\left\{\frac{d}{dt} f(t)\right\} = sF(s) - f(0)$
- Integration: $L\{\int f(t)dt\} = \frac{1}{s} F(s)$
- Displacement: $L\{f(t-\tau)\} = e^{-\tau s} F(s)$
- $L\{e^{\alpha t}\} = \frac{1}{s-\alpha}$ $L\{t^n\} = \frac{n!}{s^{n+1}} \quad (n = 1,2 \ \dots)$
- $L\{\sin(\alpha t)\} = \frac{\alpha}{s^2 + \alpha^2}$ $L\{cos(\alpha t)\} = \frac{s}{s^2 + \alpha^2}$

Laplace Transform Table

Time Domain	Laplace Domain
$f(t)$	$\mathcal{L}\big(f(t)\big) = F(s)$
$f(t), g(t)$	$F(s), G(s)$
1	$1/s$
$e^{\alpha t}$	$1/(s-\alpha)$
$t^n e^{\alpha t}, n = 1,2, \ldots$	$n!/(s-a)^{n+1}$
t^n	$n!/s^{n+1}, n = 1,2, \ldots$
$t^{-\frac{1}{2}}$	$\sqrt{\pi/s}$
$\sin(\alpha t)$	$\alpha/(s^2+\alpha^2)$
$\cos(\alpha t)$	$s/(s^2+\alpha^2)$
$\sinh(kt)$	$k/(s^2-k^2)$
$\cosh(kt)$	$s/(s^2-k^2)$

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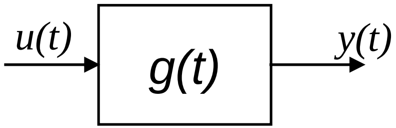
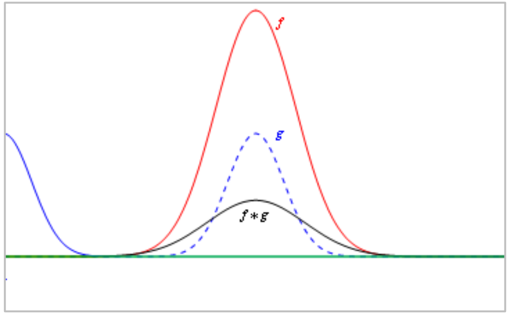
Transfer Elements and Transfer Function

- Linear time-invariant transfer element (LTI element)





wikipedia

- In the complex s-domain:

$$Y(s) = G(s) \cdot U(s)$$

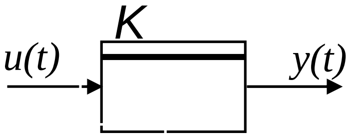
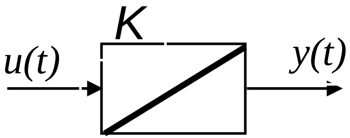
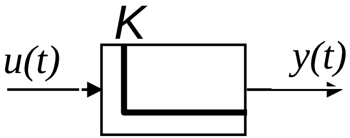
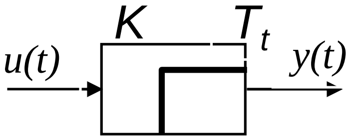
$G(s)$: Transfer function

Convolution rule of the Laplace transformation

- In the time domain:

$$y(t) \quad = \quad g(t) * u(t) = \int_0^t g(\tau) \cdot u(t-\tau) d\tau, \ \text{für } t \geq 0$$

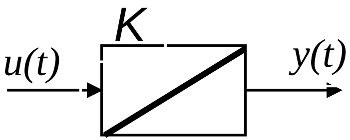
Elementary Transfer Element (1)

Name	Functional Relationship	Symbol
P-Element Proportional Element	$y(t) = K \cdot u(t)$	
I-Element Integral Element	$y(t) = K \cdot \int_0^t u(\tau)d\tau$	
D-Element Derivative Element	$y(t) = K \cdot \dot{u}(t)$	
T_t-Element Time Delay Element (Dead time Element)	$y(t) = K \cdot u(t - T_t)$	

Transfer Function of I-Element

I-Element
Integral Element

$$y(t) = K \cdot \int_0^t u(\tau) d\tau$$

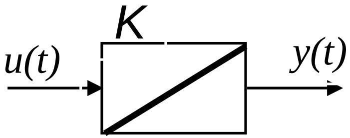


Transfer Function of I-Element

I-Element

Integral Element

$$y(t) = K \cdot \int_0^t u(\tau) d\tau$$



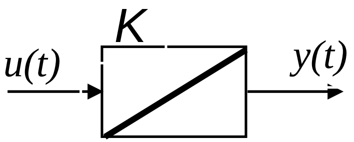
Laplace:

Transfer Function of I-Element

I-Element

Integral Element

$$y(t) = K \cdot \int_0^t u(\tau)d\tau$$



Laplace:

$$Y(s) = K \cdot \frac{1}{s} \cdot U(s) = \underbrace{\frac{K}{s}}_{G(s)} \cdot U(s)$$

Example: $u(t) = \sigma(t)$, $\mathrm{U}(s) = \frac{1}{s}$ (Step function)

$$Y(s) = \frac{K}{s} \cdot \frac{1}{s} = \frac{K}{s^2} \Rightarrow y(t) = K \cdot t$$

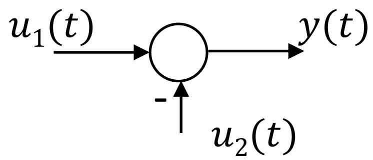
$f(t) = t^n$	$F(s) = n!/s^{n+1}$ $n = 1,2,\ldots$

Elementary Transfer Element (2)

Name | **Functional Relationship** | **Symbol**

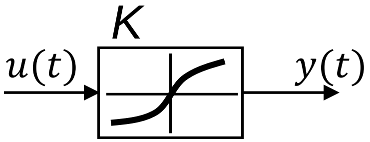
S-Element
Summing Element

$$y(t) = \pm u_1(t) \pm u_2(t)$$



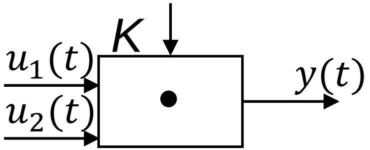
Ch-Element
Characteristic Element

$$y(t) = K \cdot f(u(t))$$



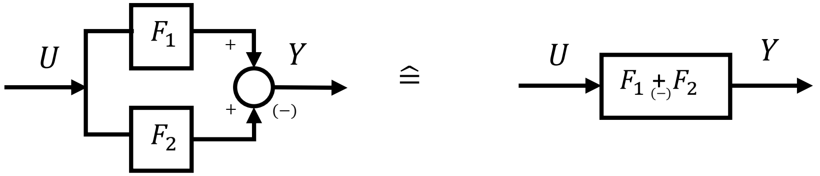
M-Element
Multiplication Element

$$y(t) = K \cdot u_1(t) u_2(t)$$



Transfer Element: Rules

Transformation rules for block diagrams

1) 

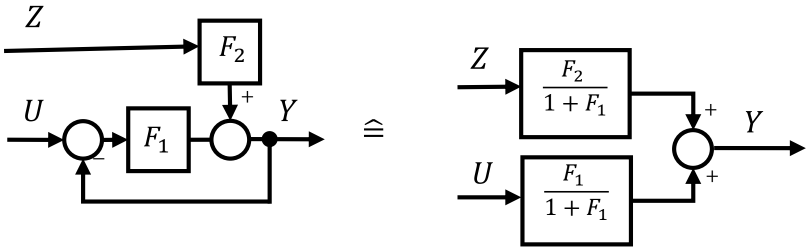
$$Y(s) = (F_{1(s)} \pm F_{2(s)})U(s)$$

2) 

$$Y(s) = (F_{1(s)} \cdot F_{2(s)})U(s)$$

3) 

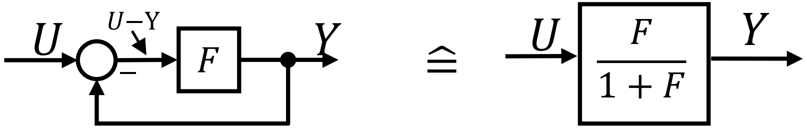
$$Y(s) = \left(\frac{F(s)}{1 + F(s)}\right)U(s)$$

4) 

$$Y(s) = \frac{F_1(s)}{1 + F_1(s)} \cdot U(s) + \frac{F_2(s)}{1 + F_1(s)} \cdot Z(s)$$

Transfer Element: Rules

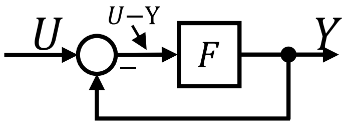
Transformation rules for block diagrams

3) 

$$Y(s) = \left(\frac{F(s)}{1 + F(s)}\right) U(s)$$

Transfer Element: Rules

Transformation rules for block diagrams

3) 

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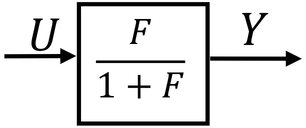


$$Y(s) = \left(\frac{F(s)}{1+F(s)}\right)U(s)$$

$$Y(s) = F(s) \cdot \big(U(s) - Y(s)\big)$$

$$Y(s) + F(s) \cdot Y(s) = F(s) \cdot U(s)$$

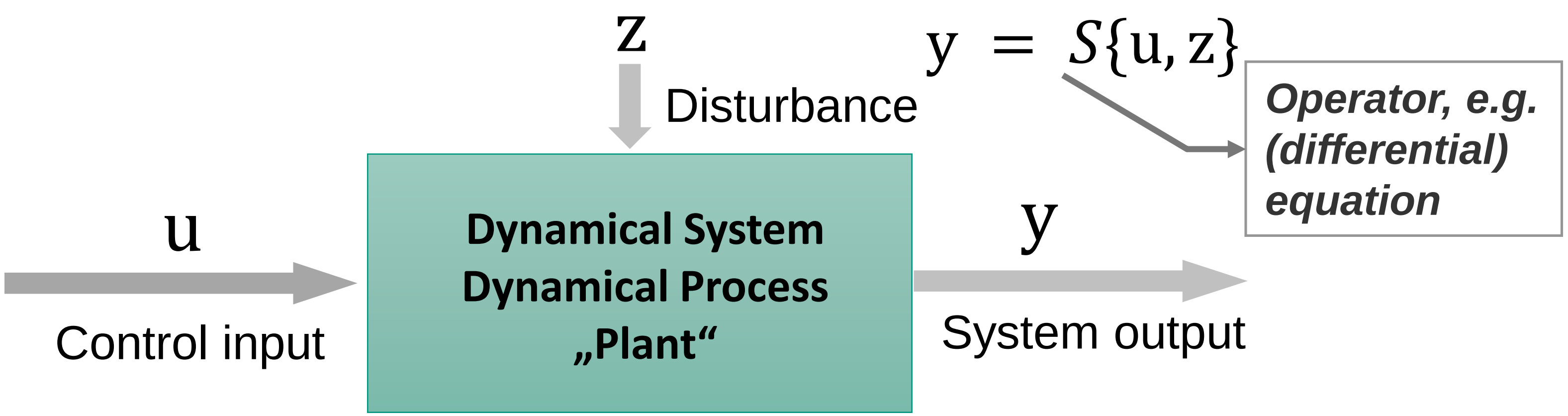
$$Y(s) \cdot \big(1 + F(s)\big) = F(s) \cdot U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{F(s)}{1+F(s)}$$

Recap

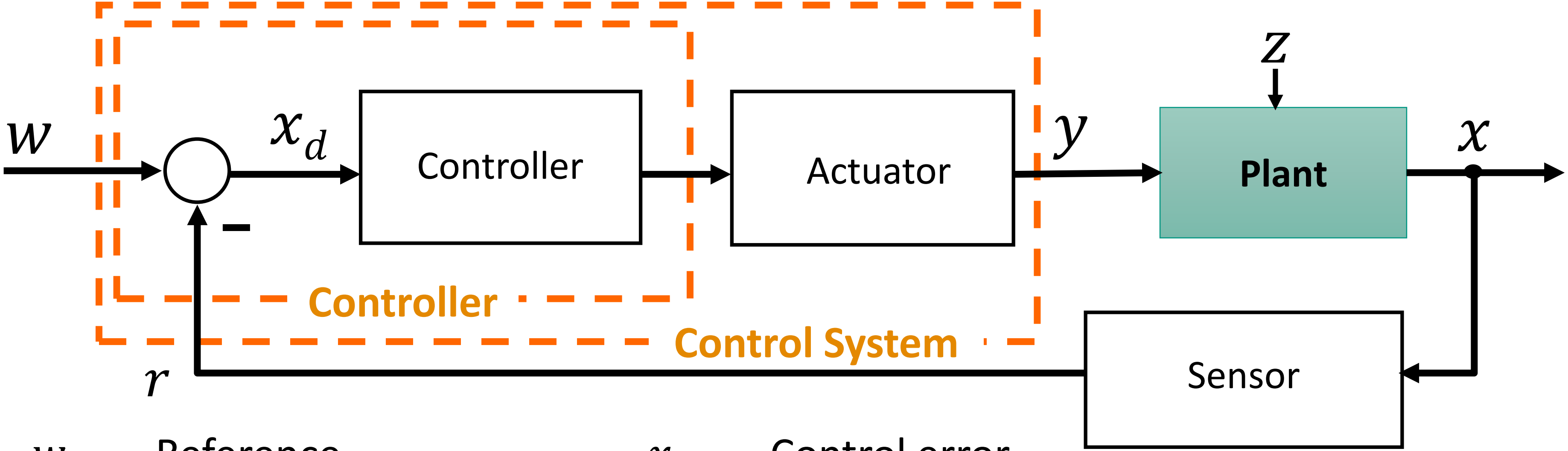
- Definition of control system
- Laplace transform
- Transfer function
- Transfer element

Recap: Structure and Operation of a Control System



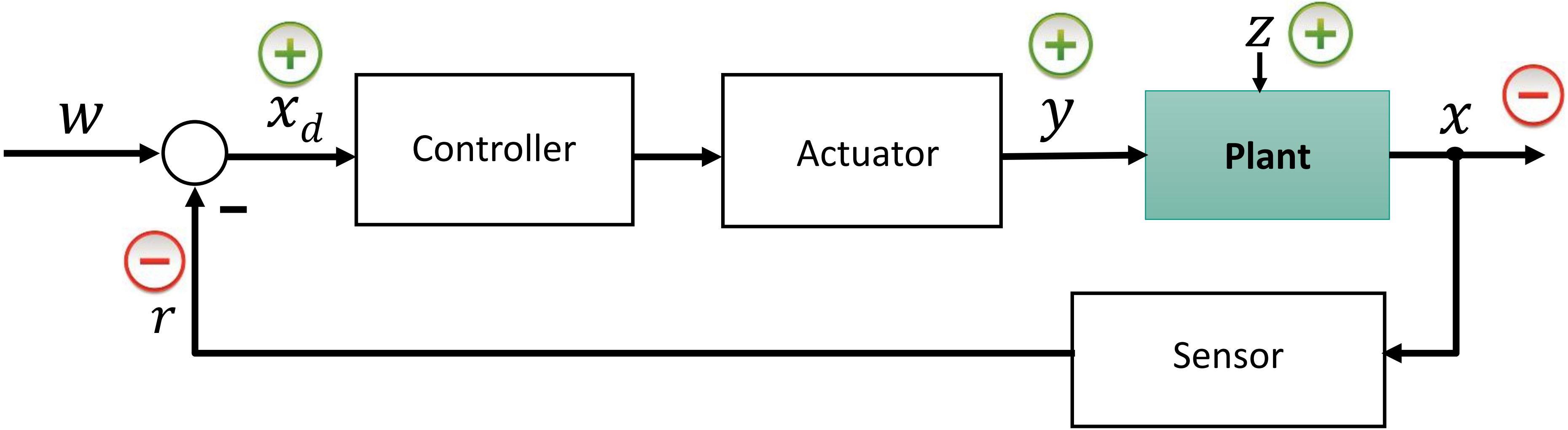
- **Task:**
  The system output is to be influenced via the control input in such a way that a desired system behavior (i.e. system output) is achieved, despite a disturbance that is not or only partially known

Recap: Structure of a Control System



w	Reference	x_d	Control error
y	Control Input	x	System output
r	Feedback	z	Disturbance

Recap: Structure of a Control System



Target value of x: x_s

Measurement: $r = K_j\, x \qquad K_j > 0$ (constant)

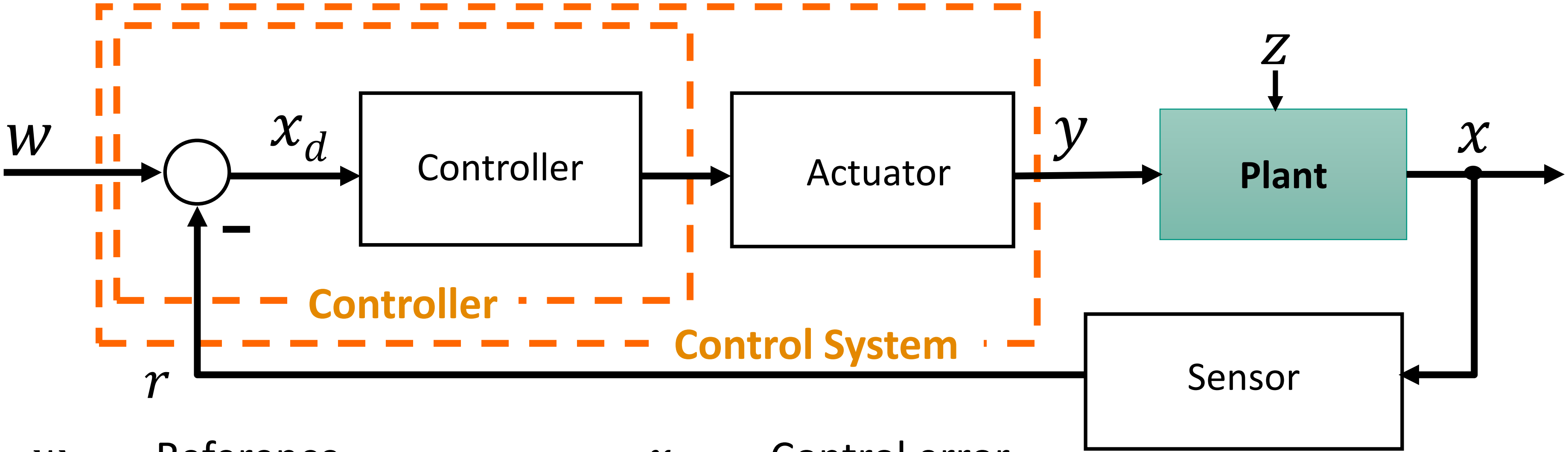
Selection of reference: $w = K_j\, x_s$

Then: $x_d = w - r = K_j\, x_s - K_j\, x = K_j\,(x_s - x)$

Creating the Diagram of the Control Loop

- From physical laws, we can derive **equations (differential or difference equations) that describe the relationships between time-varying quantities of the system**.
- The time-varying quantities and their equations are represented by suitable symbols.
- A block in the block diagram uniquely assigns each time response of the input variable to a time respones of the output variable, thus acting as a transfer element.

Structure of a Control System



w	Reference	x_d	Control error
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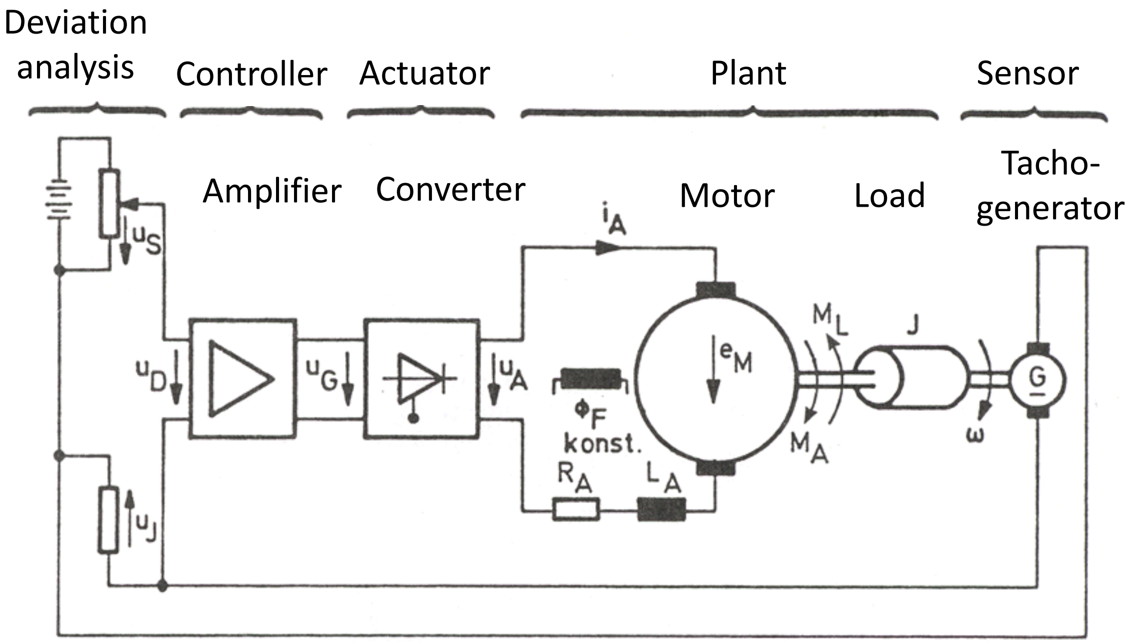
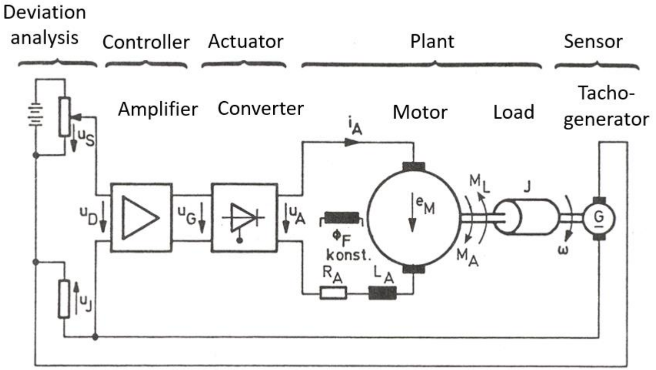
Example: Velocity Control of a DC Motor



German original taken from: ***Regelungstechnik; O. Föllinger***

Physical Laws: Velocity Control Equations



- Armature circuit of the motor

$u_R = u_A - e_M$ e_M = Back Electromotive Force (back EMF)

$e_M = K_F \cdot \omega$ K_F = Field constant

$$u_R = u_A - e_M = R_A \cdot i_A + L_A \cdot \dot{i}_A \quad \rightarrow \quad \frac{L_A}{R_A} \dot{i}_A + i_A = \frac{1}{R_A} u_A$$

- Mechanical movement of the armature under load

$J \cdot \dot{\omega} = M_E$ J = Moment of inertia of armature and load

M_E = Effective torque

$M_E = M_A - M_L$ M_A = Armature torque

M_L = Load torque of the motor

$M_A = K_F \cdot i_A$

- Power converter: $T_{ST} \cdot \dot{u}_A + u_A = K_{ST} \cdot u_G$
- Electrical amplifier: $u_G = K_V \cdot u_D$
- Speedometer generator: $u_J = K_J \cdot \omega$

Transform D-Eq. into Ordinary Equations

- Armature circuit of the motor

$$u_R = u_A - e_M$$
$$e_M = K_F \cdot \omega$$

e_M = Opposing EMF
K_F = Field constant

$$u_R = u_A - e_M = R_A \cdot i_A + L_A \cdot \dot{i}_A \quad \rightarrow \quad \frac{L_A}{R_A}\dot{i}_A + i_A = \frac{1}{R_A} u_A$$

- Mechanical movement of the armature under load

$$J \cdot \dot{\omega} = M_E$$

J = Moment of inertia of armature and load
M_E = Effective torque

$$M_B = M_A - M_L$$

M_A = Armature torque
M_L = Load torque of the motor

$$M_A = K_F \cdot i_A$$

- Power converter: $T_{ST} \cdot \dot{u}_A + u_A = K_{ST} \cdot u_G$
- Electrical amplifier: $u_G = K_V \cdot u_D$
- Speedometer generator: $u_J = K_J \cdot \omega$

Differential Equations (D-Eq.)

How to deal with this?

With Laplace Transform

Laplace Transform

$$\frac{L_A}{R_A}\,\dot{i}_A + i_A = \frac{1}{R_A}u_R$$

$$\Big\downarrow \mathcal{L}$$

$$\mathcal{L}[\dot{f}(t)] = s\int_0^\infty e^{-st} f(t)dt - f(0) = s \cdot F(s) - f(0)$$

$$\mathcal{L}[\dot{i}_A\,(t)] = s\,I_A(s) - i_A(0) \qquad i_A(0) = 0$$

Laplace Transform

$$\frac{L_A}{R_A}\, \dot{i}_A + i_A = \frac{1}{R_A} u_R$$

$$\downarrow \mathcal{L}$$

$$\frac{L_A}{R_A}\, s \cdot I_A(s) + I_A(s) = \frac{1}{R_A} U_R(s)$$

$$\left(\frac{L_A}{R_A} \cdot s + 1\right) \cdot I_A(s) = \frac{1}{R_A} U_R(s) \rightarrow I_A(s) = \frac{\frac{1}{R_A}}{1+\frac{L_A}{R_A} \cdot s} \cdot U_R(s)$$

$$\mathcal{L}[\dot{f}(t)] = s \int_0^\infty e^{-st} f(t)dt - f(0) = s \cdot F(s) - f(0)$$

$$\mathcal{L}[\dot{i}_A(t)] = s\, I_A(s) - i_A(0) \qquad i_A(0) = 0$$

$$I_A(s) = G_1(s) \cdot U_R(s)$$

$G_1(s)$: Transfer function

Laplace Transform

$$J \cdot \dot{\omega} = M_B$$

$$\omega\,(t) = \int \frac{1}{J} \cdot M_B(t)\,dt$$

$\downarrow \mathcal{L}$

$$\omega\,(s) = \frac{1}{J} \cdot \frac{1}{s} \cdot M_B\,(s)$$

$$\omega(s) = G_2(s) \cdot M_B(s)$$

$$\mathcal{L}\left[\int_0^t f(t)\,dt\right] = \frac{1}{s}\,F(s)$$

Laplace Transform

Similarly

$$\mathcal{L}[\dot{f}(t)] = s\int_0^{\infty} e^{-st} f(t)dt - f(0) = s \cdot F(s) - f(0)$$

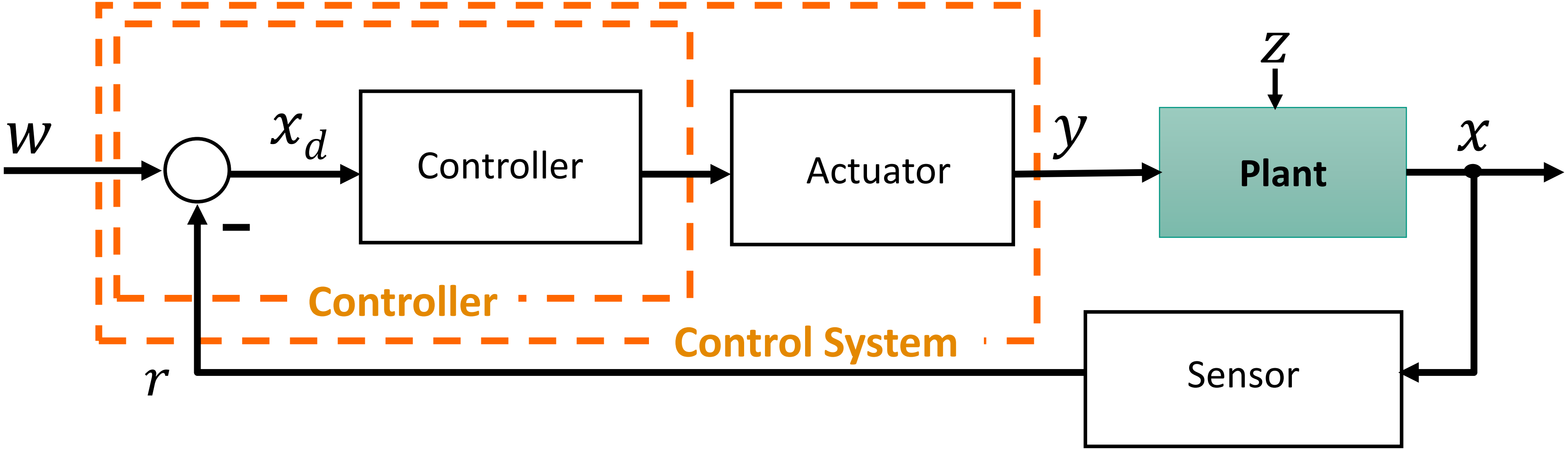
$$T_{ST} \cdot \dot{u}_A + u_A = K_{ST} \cdot u_G$$

$$\downarrow \mathcal{L}$$

$$U_A(s) = \underbrace{\frac{K_{ST}}{1 + T_{ST} \cdot s}} \cdot U_G(s)$$

$$U_A(s) = G_3(s) \cdot U_G(s)$$

Structure of a Control System



w	Reference	x_d	Control error
y	Control Input	x	System output
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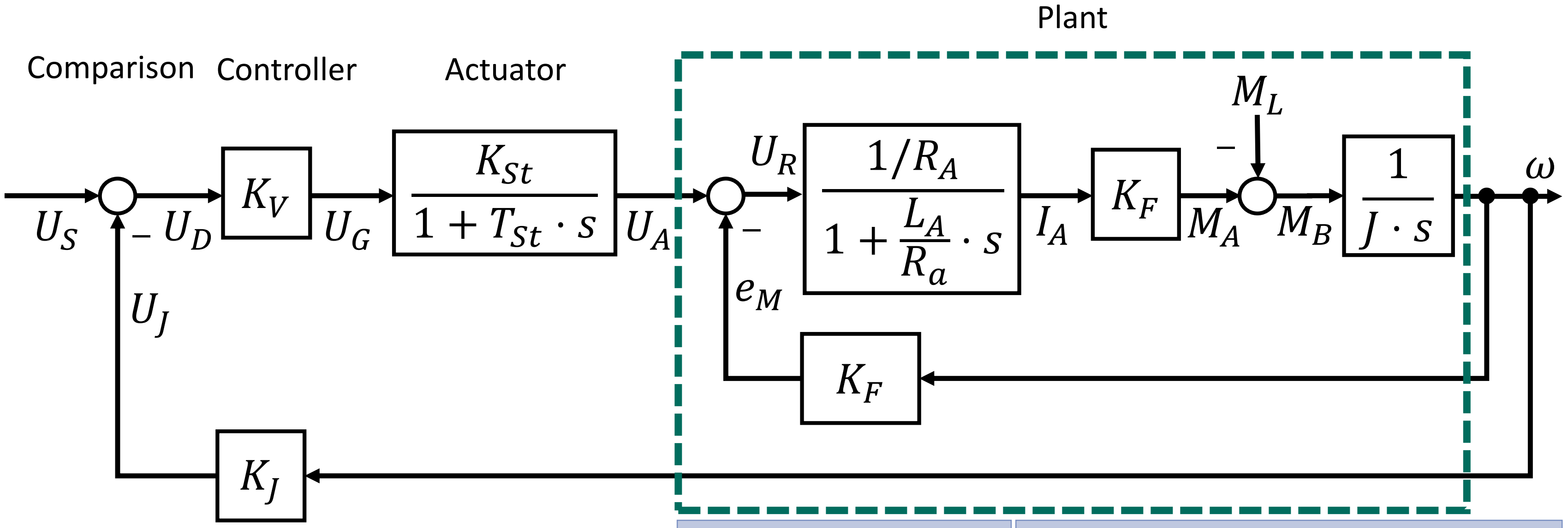
Velocity Controller of a DC Motor



Measurement

Controller

$$\omega(s) = \frac{1}{J}\cdot\frac{1}{s}\cdot M_B(s)$$
$$M_B = M_A - M_L$$
$$M_A = K_F \cdot I_A$$
$$I_A(s) = \frac{\frac{1}{R_A}}{1+\frac{L_A}{R_A}\cdot s}\cdot U_R(s)$$

$$U_R = U_A - e_M$$
$$e_M = K_F\cdot\omega$$
$$U_A(s) = \frac{K_{ST}}{1+T_{ST}\cdot s}\cdot U_G(s)$$
$$U_G = K_V\cdot U_D$$
$$U_D = U_s - U_J$$
$$U_J = K_J\cdot\omega$$

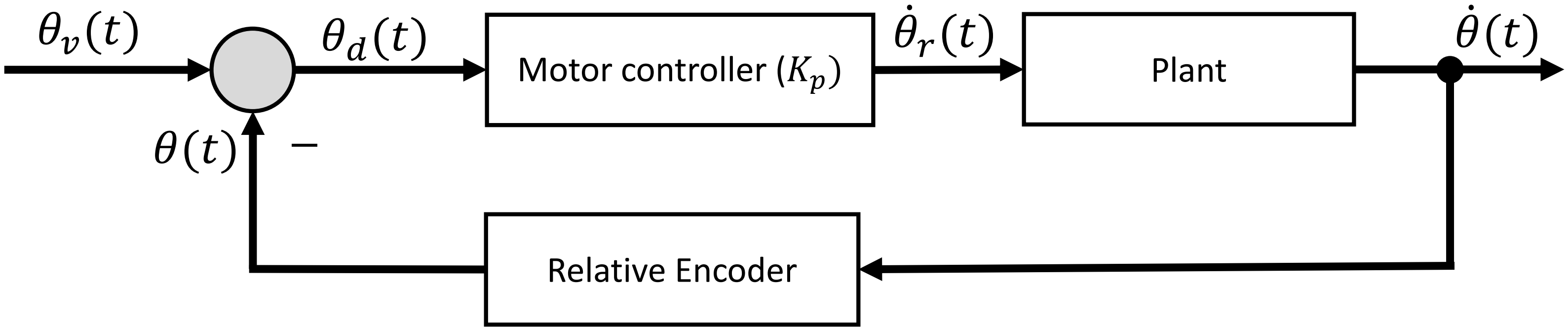
Contents

Velocity Control

- In the joint space: Continuous specification of joint velocities
- **Proportional control** with factor K_p

$$\dot{\theta}_r(t) = K_p \cdot \left(\theta_v(t) - \theta(t)\right)$$

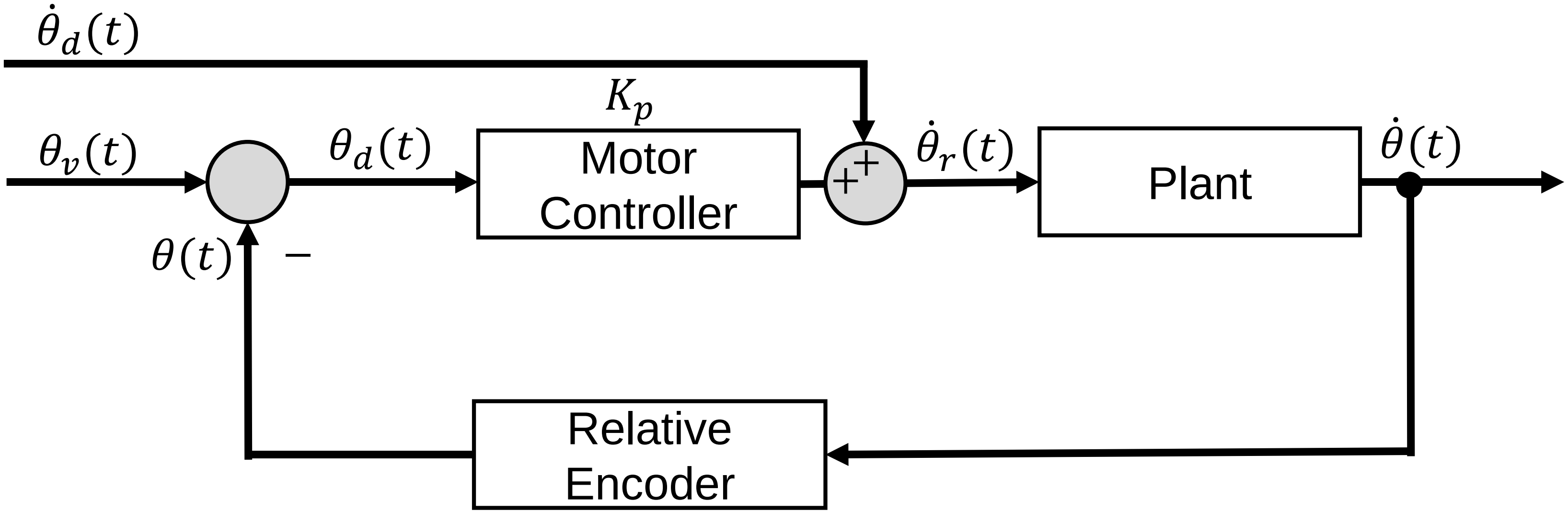
- Property: if $\theta_d = 0$, the joint does not move.



Feedforward Control

- Velocity specificiation even if $\theta_d = 0$.

$$\dot{\theta}_r(t) = K_p \cdot \big(\theta_v(t) - \theta(t)\big) + \dot{\theta}_d(t)$$

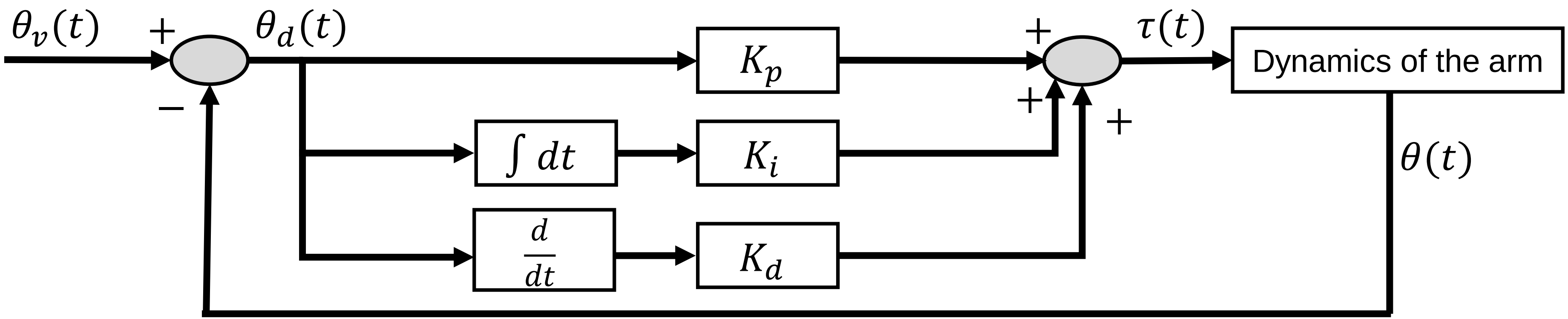


PID-Controller

- **Proportional-Integral-Derivative** Controller

$$\tau(t) = K_p\theta_d(t) + K_i\int \theta_d(t)dt + K_d\dot{\theta}_d(t)$$

- K_p: “virtual spring” that reduces the position error
- K_d: “virtual damper” that reduces the speed error
- K_i: reduces control deviations (offsets)



Laplace Transform of the PID-Controller

$$\tau(t) = K_P\theta_d(t) + K_I \int \theta_d(t)dt \; + K_D \frac{d}{dt}\theta_d(t)$$

$$\Big\downarrow \; \mathcal{L}$$

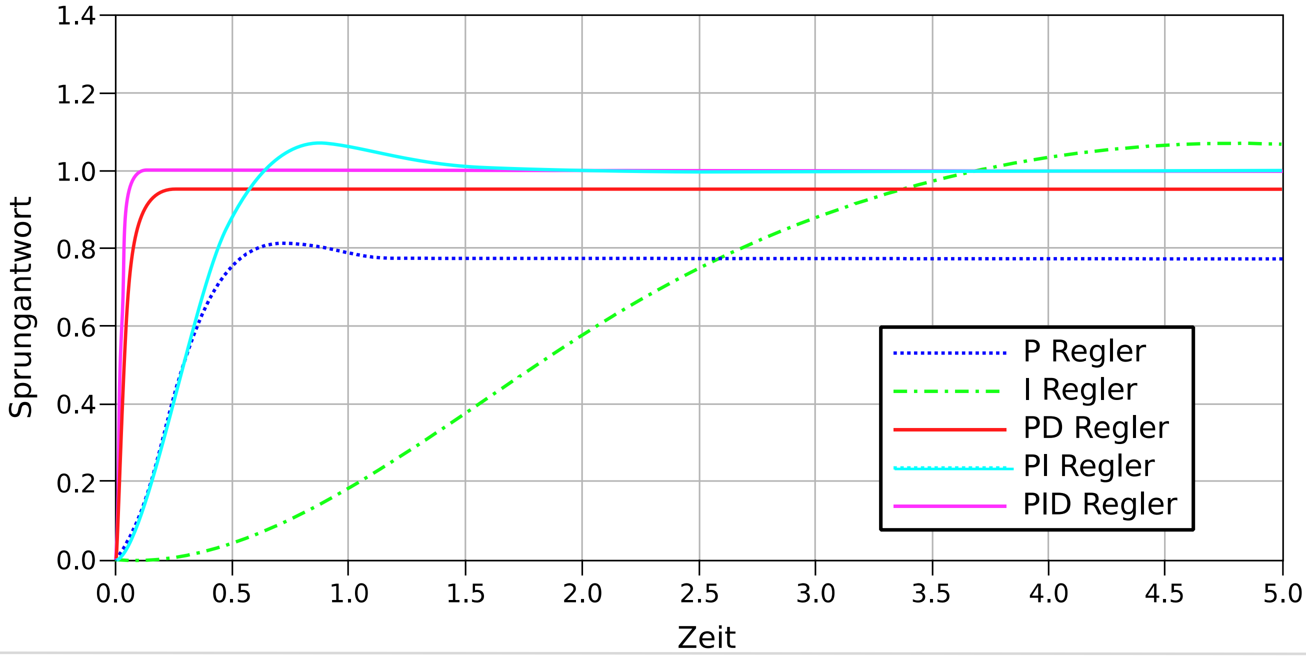
$$\tau(s) = K_P \cdot \theta_d(s) + K_I \frac{1}{s} \cdot \theta_d(s) + K_D s \cdot \theta_d(s)$$

$$\frac{\tau(s)}{\theta_d(s)} = G(s) = K_P + K_I \frac{1}{s} + K_D s$$

$$\frac{\text{Output}}{\text{Input}} = \text{Transfer function}$$

PID-Controller

Comparison of P-, I-, PD-, PI- und PID-controllers in a control loop with PT2-element as controlled system (linear time-invariant 2nd order delay element)



I-component:
Compensate for control deviations (steady-state Accuracy

D-component:
Dynamics (how fast)

Classic Controller Types

- PID-controller (and subclasses)
  - Very common, due to being suitable for almost all process types, robust and can be realized with little effort
  - **Characteristic equation:**

$$u(t) = K_p\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{d}{dt}e(t)\right)$$

    - P-component: favorable control characteristics
    - I-component: steady-state accuracy
    - D-component: fast regulation

with T_N = integral time, T_V = derivative time

Contents

Example: 1-DoF Torque Control

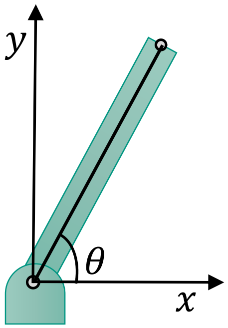
- The robot **dynamic model** is considered in the control system
- Dynamic equation for 1-DoF arm
  (planar rotation, no gravity):

$$\tau = M\ddot{\theta} + b\dot{\theta}$$

τ: Torque of the motor
M: Inertia tensor
b: Friction

- **Goal:**
  Fixed-point controller (keep value constant)
  realized as PD controller

  Setpoint (target value): $\boldsymbol{\theta_v = const}$



- PD-Controller

$$\tau = K_p\theta_d + K_d\dot{\theta}_d$$

Stability: 1-DoF Torque Control (1)

- System (Plant)

$$\tau = M\ddot{\theta} + b\dot{\theta}$$

- Controller

$$\tau = K_p\theta_d + K_d\dot{\theta}_d$$

- Control error $\theta_d = \theta_v - \theta \qquad (\theta_v = const)$

$$\theta = \theta_v - \theta_d, \qquad \dot{\theta} = -\dot{\theta}_d, \qquad \ddot{\theta} = -\ddot{\theta}_d$$

Stability: 1-DoF Torque Control (1)

- System (Plant)

$$\tau = M\ddot{\theta} + b\dot{\theta}$$

- Controller

$$\tau = K_p\theta_d + K_d\dot{\theta}_d$$

- Relevant for us: Control error $\quad \theta_d = \theta_v - \theta \qquad (\theta_v = const)$

$$\theta = \theta_v - \theta_d, \qquad \dot{\theta} = -\dot{\theta}_d, \qquad \ddot{\theta} = -\ddot{\theta}_d$$

- Equating gives differential equation:

$$K_p\theta_d + K_d\dot{\theta}_d = M\ddot{\theta} + b\dot{\theta}$$
$$K_p\theta_d + K_d\dot{\theta}_d = -M\ddot{\theta}_d - b\dot{\theta}_d$$
$$M\ddot{\theta}_d + (K_d + b)\dot{\theta}_d + K_p\theta_d = 0$$

$$\ddot{\theta}_d + \frac{(K_d + b)}{M} \cdot \dot{\theta}_d + \frac{K_p}{M} \cdot \theta_d = 0$$

2. Order Differential Eq.: Can be solved with the help of the Laplace transform

Stability: 1-DoF Torque Control (Calculation)

$$\theta_v = const \qquad \dot{\theta}_v = 0 \qquad \ddot{\theta}_v = 0$$

$$\theta_d = \theta_v - \theta \qquad \dot{\theta}_d = -\dot{\theta} \qquad \ddot{\theta}_d = -\ddot{\theta}$$

$$M\ddot{\theta} + b\dot{\theta} = K_p(\theta_v - \theta) + K_d(\dot{\theta}_v - \dot{\theta})$$

$$M(-\ddot{\theta}_d) + b(-\dot{\theta}_d) = K_p\theta_d + K_d\left(0 - (-\dot{\theta}_d)\right)$$

$$-M\ddot{\theta}_d - b\dot{\theta}_d = K_p\theta_d + K_d\dot{\theta}_d$$

$$-M\ddot{\theta}_d - b\dot{\theta}_d - K_d\dot{\theta}_d - K_p\theta_d = 0 \qquad | -K_d\dot{\theta}_d - K_p\theta_d$$

$$-M\ddot{\theta}_d - (K_d + b)\dot{\theta}_d - K_p\theta_d = 0 \qquad | \cdot \left(-\frac{1}{M}\right)$$

$$\ddot{\theta}_d + \frac{(K_d+b)}{M}\dot{\theta}_d + \frac{K_p}{M}\theta_d = 0$$

Stability: 1-DoF Torque Control (2)

- Description of the system with D-Eq.:

$$\ddot{\theta}_d + \frac{(K_d + b)}{M} \cdot \dot{\theta}_d + \frac{K_p}{M} \cdot \theta_d = 0$$

- Harmonic oscillation:

$$\ddot{\theta}_d + 2\zeta\omega_n\dot{\theta}_d + \omega_n^2\,\theta_d = 0$$

  - ζ: Damping
  - ω_n: Natural frequency

- For 1-DoF torque control:

$$\zeta = \frac{b + \boldsymbol{K_d}}{2\sqrt{\boldsymbol{K_p} M}}, \qquad \omega_n = \sqrt{\frac{\boldsymbol{K_p}}{M}}$$

Stability: 1-DoF Torque Control (3)

- Harmonic oscillation:

$$\ddot{\theta}_d + 2\zeta\omega_n\dot{\theta}_d + \omega_n^2\,\theta_d = 0$$

- Laplace transform:

Stability: 1-DoF Torque Control (3)

- Harmonic oscillation:

$$\ddot{\theta}_d + 2\zeta\omega_n\dot{\theta}_d + \omega_n^2\,\theta_d = 0$$

- Laplace transform:

$$s^2 \cdot \mathcal{L}[\theta_d] + 2\zeta\omega_n \cdot s \cdot \mathcal{L}[\theta_d] + \omega_n^2 \cdot \mathcal{L}[\theta_d] = 0$$
$$(s^2 + 2\zeta\omega_n \cdot s + \omega_n^2) \cdot \mathcal{L}[\theta_d] = 0$$

- Two solutions (apart from the trivial solution $\mathcal{L}[\theta_d] = 0$)

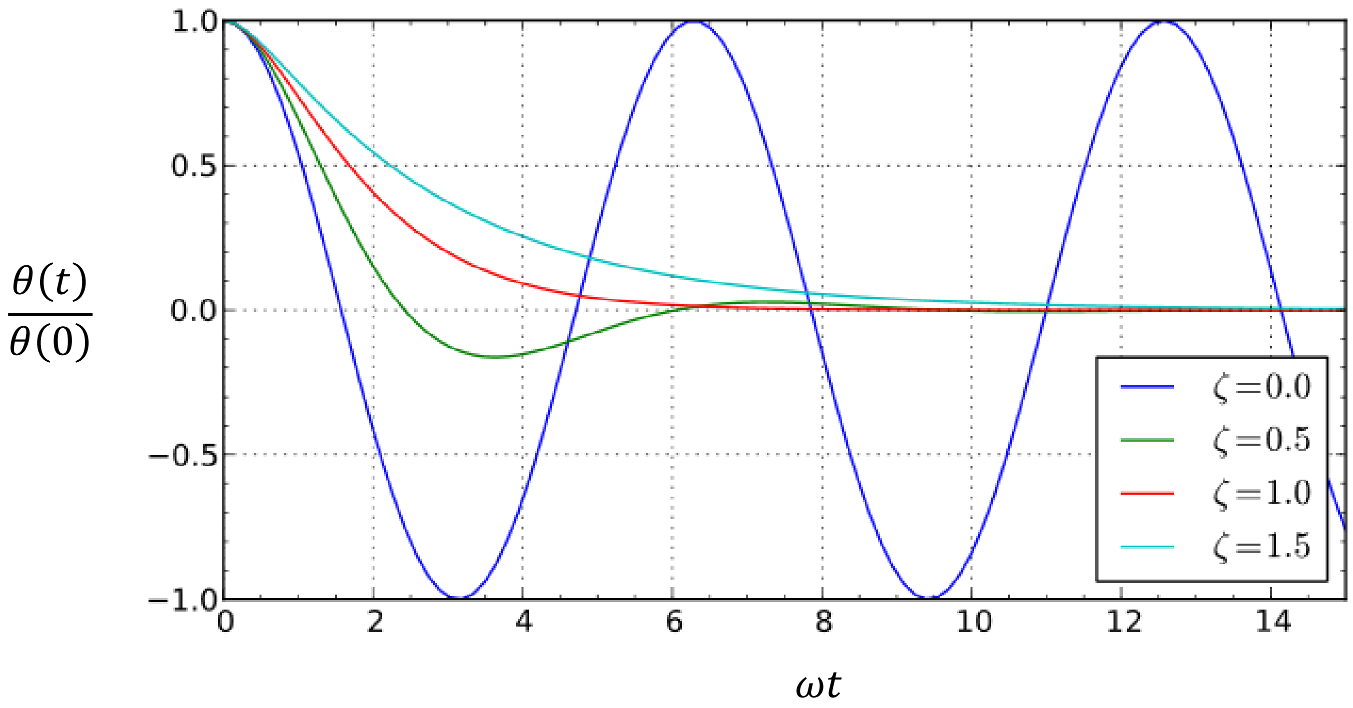
$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

Stability: 1-DoF Torque Control (4)

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

- 3 possible **solution types:**
  - $\zeta > 1$: **aperiodic solution:** two different real solutions
    $$\theta_d(t) = c_1 e^{s_1 t} + c_2 e^{s_2 t}$$
    Target value is (slowly) reached via the exponential function without overshooting
  - $\zeta = 1$: **critically damped response:** two identical real solutions ($s_{1,2} = -\zeta\omega_n$)
    $$\theta_d(t) = (c_1 + c_2 t)e^{-\zeta\omega_n t}$$
    The target value is reached quickly and the system just does not overshoot
  - $\zeta < 1$: **damped oscillation:** two complex solutions
    $$\theta_d(t) = (c_1 \cos(\omega_n t) + c_2 \sin(\omega_n t))e^{-\zeta\omega_n t}$$
    The system overshoots

Stability: 1-DoF Torque Control (5)



Stability: 1-DoF Torque Control (6)

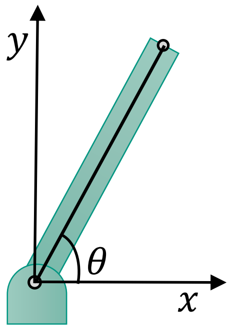
- Damping ζ is selected according to the application
- Here: No overshoot desired $\Rightarrow \zeta = 1$

$$\zeta = \frac{b + K_d}{2\sqrt{K_p M}}, \qquad \omega_n = \sqrt{\frac{K_p}{M}}$$

- Parameters for the PD controller:

$$1 = \frac{b + K_d}{2\sqrt{K_p M}} \quad \rightarrow \quad 2\sqrt{K_p M} = b + K_d$$

$$K_d = 2\sqrt{K_p M} - b$$



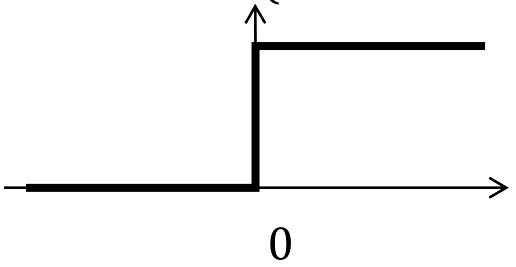
Contents

Test Functions (1)

- Impulse function
- **Step function** →
- Ramp function
- Harmonic function
- If the output variable is set to the input variable, the normalized step response is obtained $h(t)$ (transfer function of ^the system).

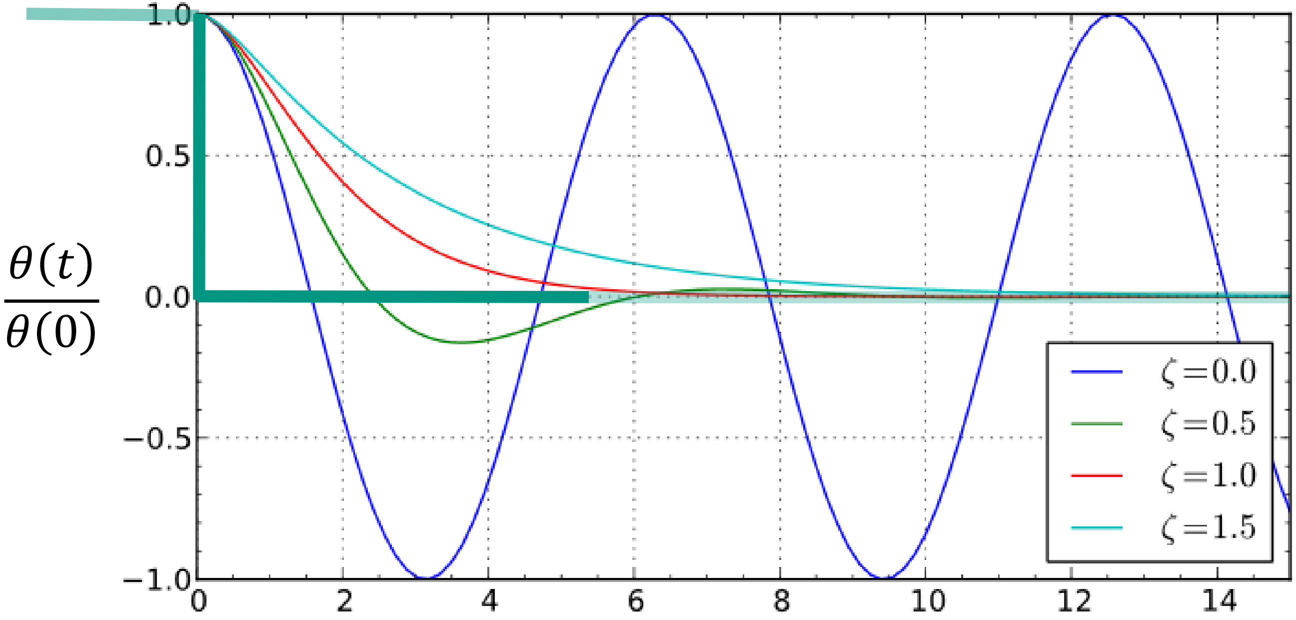
$$x_e(t) = -x_{e_0} E(t),$$

$$E(t) = \begin{cases} 0, if\ t \leq 0 \\ 1, if\ t > 0 \end{cases}$$



Test Functions (2)

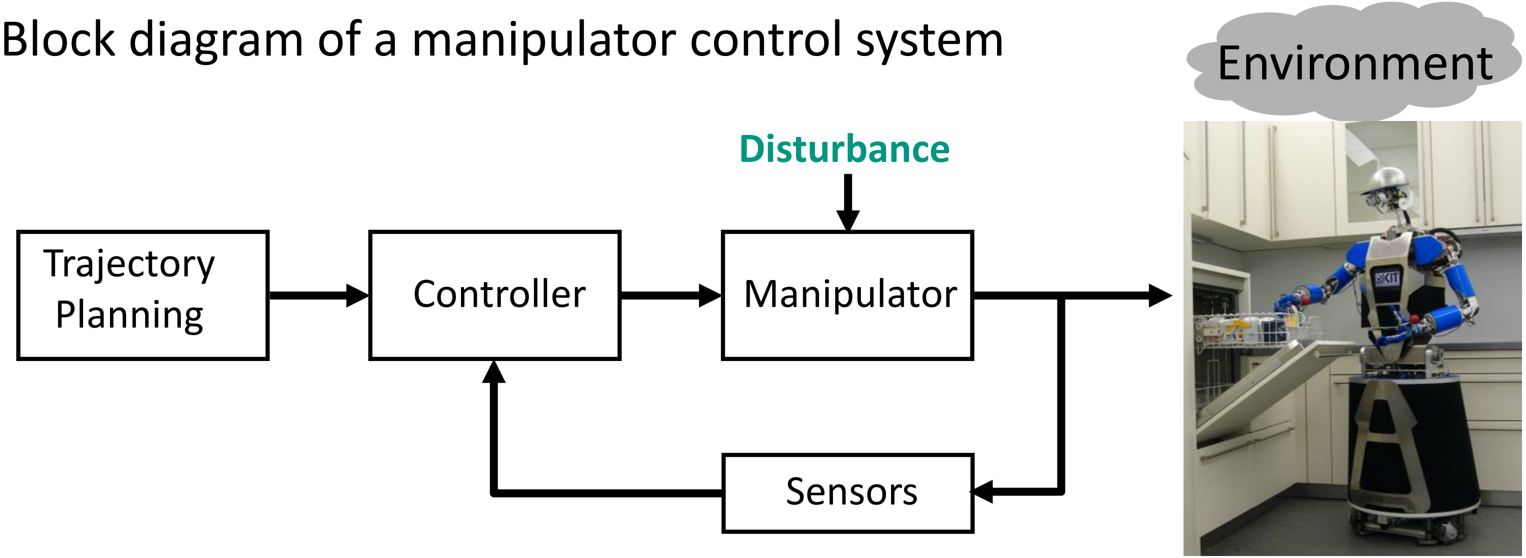
- Step function at $t = 0$



Contents

Manipulator Control

- Block diagram of a manipulator control system

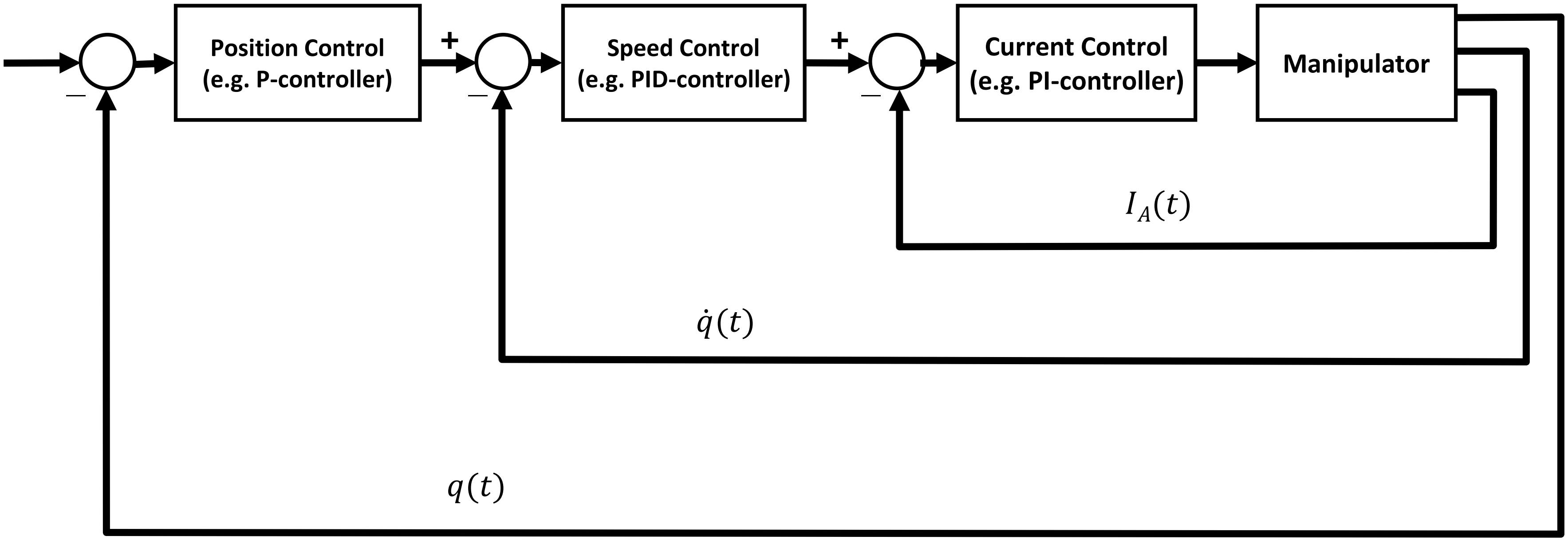


- The term “manipulator control” does not only include the classic position control, but also includes influences of the environment.
- Force and torque control plays a special role in manipulator control.

Joint-level Control: Cascading Controller

- Manipulator = **multi-body dynamic system**

  Independent linear single control loops for each individual joint



Manipulator Control

- Starting point: **dynamic model**

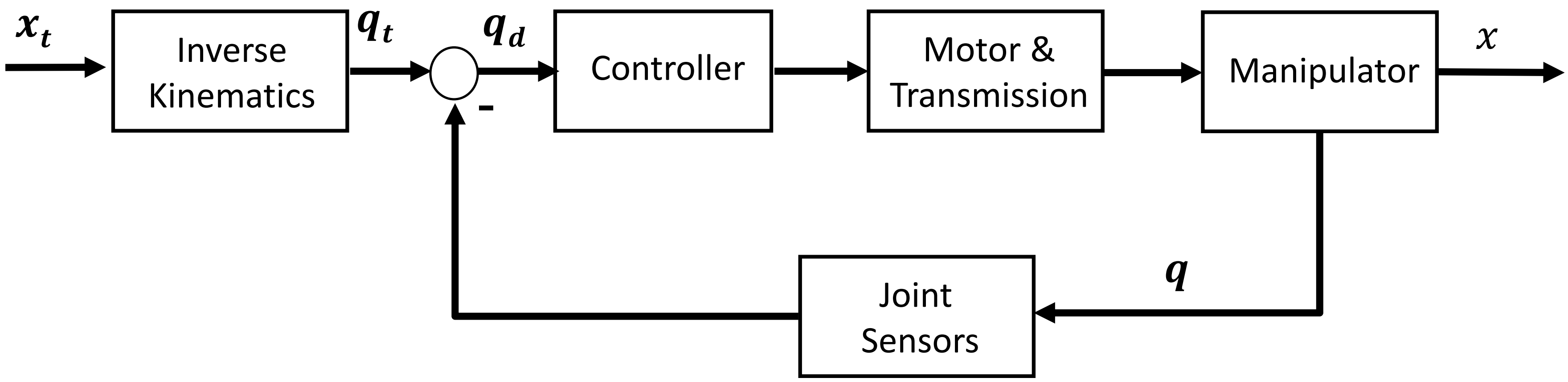
  During movements, **gravitational, centrifugal, Coriolis and frictional forces and torques** act on the joints due to the inertia of the manipulator.

$$\boldsymbol{\tau} = M(\boldsymbol{q})\ddot{\boldsymbol{q}} + n(\dot{\boldsymbol{q}}, \boldsymbol{q}) + g(\boldsymbol{q}) + R\dot{\boldsymbol{q}}$$

$\boldsymbol{\tau}$	$: n \times 1$	Vector of the general static forces and torques
$M(\boldsymbol{q})$	$: n \times n$	Inertia matrix
$\boldsymbol{n}$	$: n \times 1$	Vector with centrifugal and Coriolis components
$g(\boldsymbol{q})$	$: n \times 1$	Vector with gravitational components
R	$: n \times n$	Diagonal matrix to describe the frictional forces
$\boldsymbol{q}$	$: n \times 1$	(Generalized) Joint positions of the manipulator

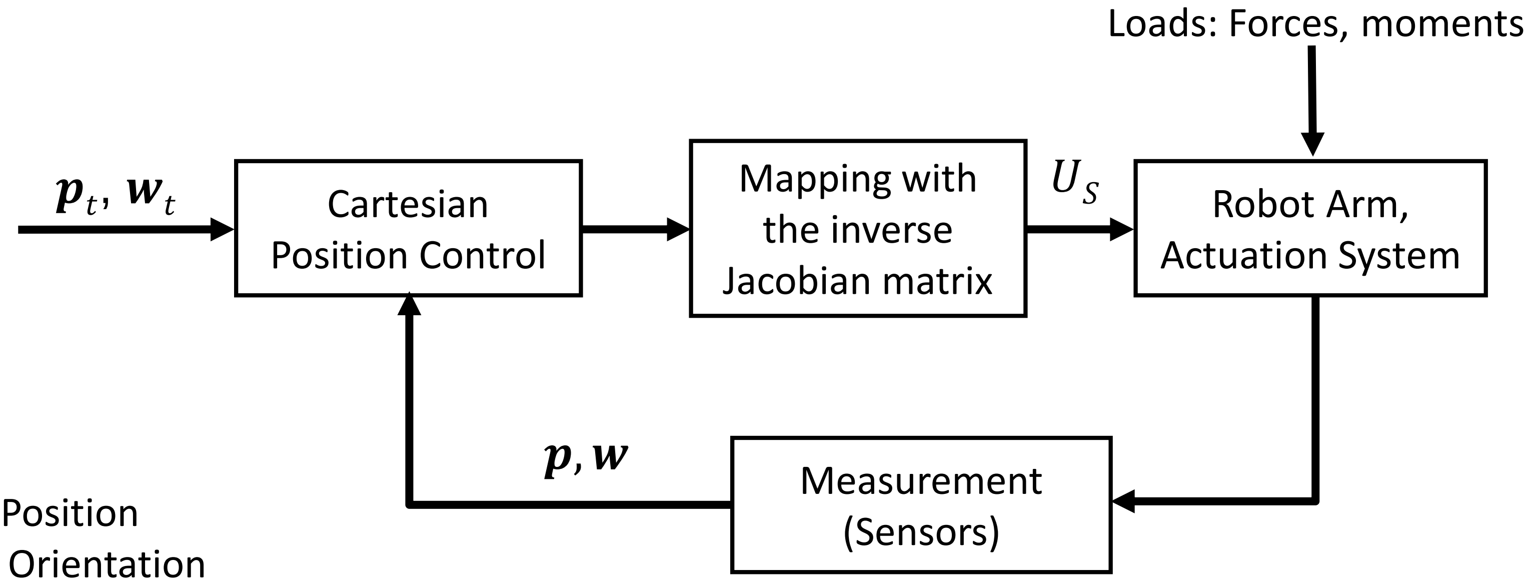
Joint Space Control

- **Coordinate transformation:** Target trajectories in joint space
- Target values for the joint actuators are calculated based on the target and measured joint angles.



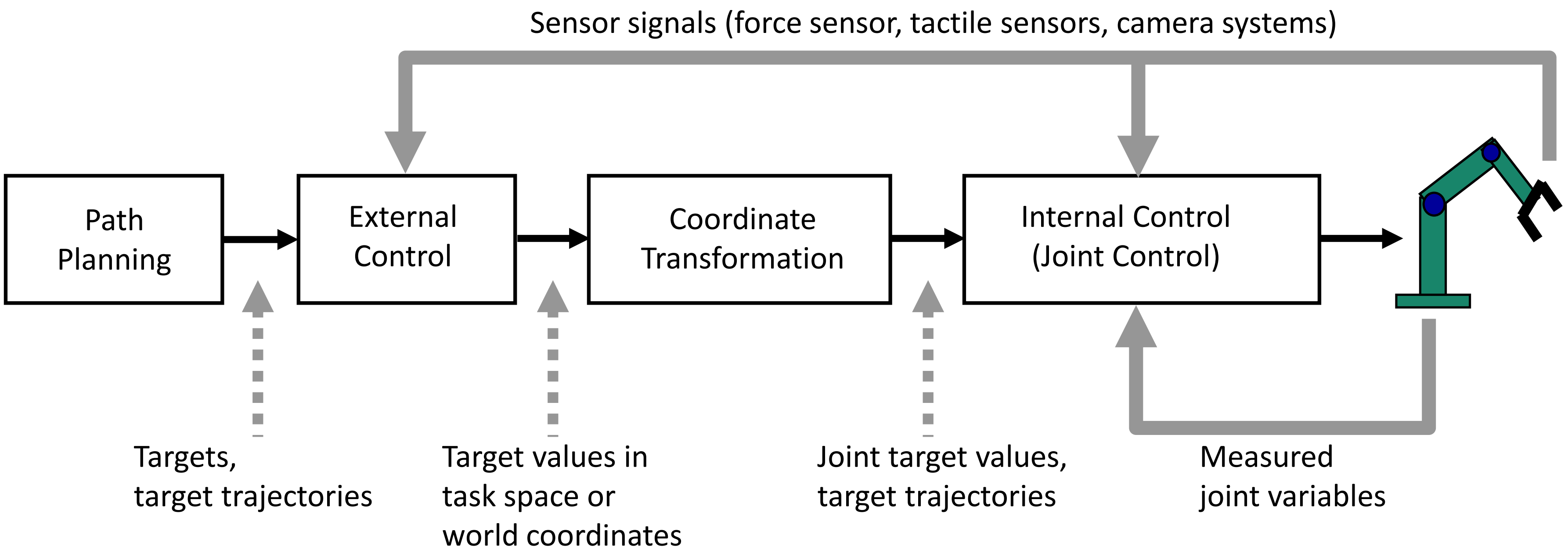
Cartesian Space Control

- **More Complexity** in the control algorithms
- Direct, targeted influencing of the individual spatial coordinates



$\boldsymbol{p}$ = Position
$\boldsymbol{w}$ = Orientation

Structure of a Robot Controller



Control Concepts for Manipulators

- **Precise System Model**
  - Assumes a-priori exact knowledge of the robot dynamics model and its environment

- **Force/Position Control**
  - For tasks requiring interaction forces, we must consider
    - Hybrid force/position control
    - Impedance control

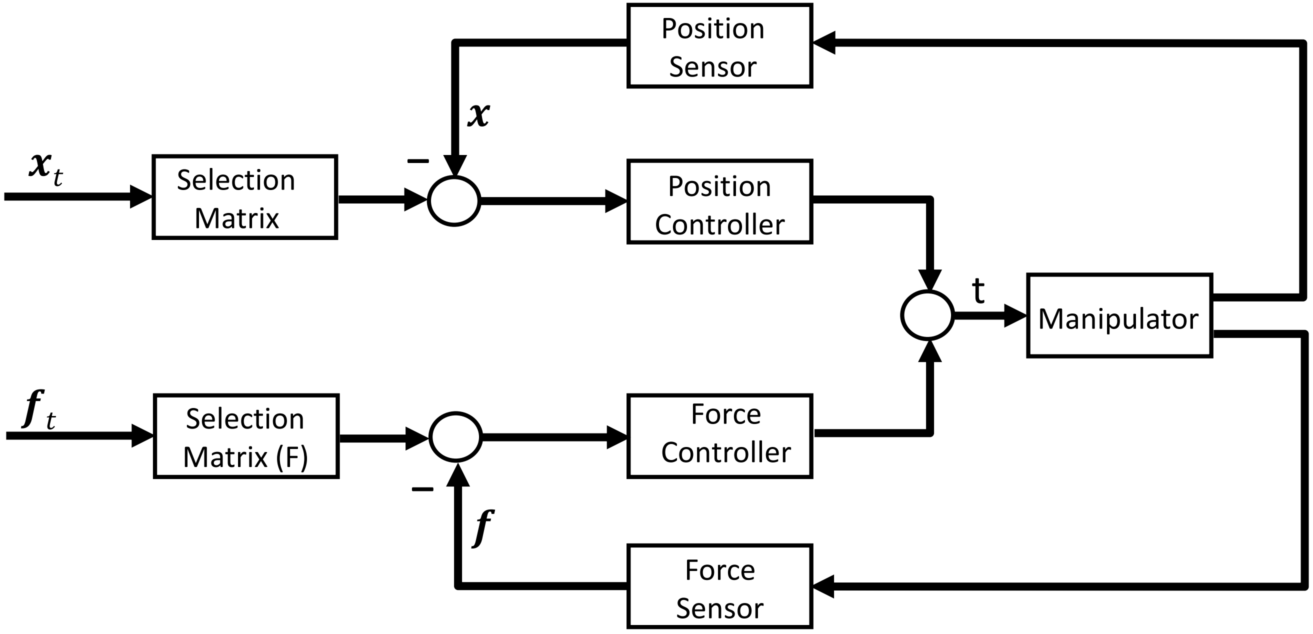
Force/Position Control

- **Fundamental Problem**
  - Positions and forces are tightly interconnected.
  - If the robot is in **contact with the environment**, every change in position also means a change in force and vice versa.

- **General method** for solving the problem
  - Derive natural boundary conditions from the description of the task to be performed. Further boundary conditions are additionally introduced to fully describe the motion.

Hybrid Force/Position Control

- Pure force or position control for each Cartesian direction of the arm movement



Impedance Control

- Control of the **dynamic relationship between force and position in case of contact.**

- **Idea:**
  - The interaction between a robot and the environment behaves like a spring-damper-mass system
  - Force f and motion (defined by: $\boldsymbol{x}(t), \dot{\boldsymbol{x}}(t), \ddot{\boldsymbol{x}}(t)$) can be calculated via the spring-damper mass equation:

$$f(t) = k \cdot \boldsymbol{x}(t) + d \cdot \dot{\boldsymbol{x}}(t) + m \cdot \ddot{\boldsymbol{x}}(t)$$

Impedance Control (2)

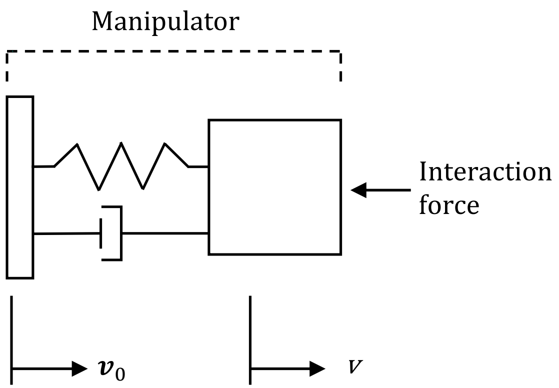
- The impedance can be influenced via stiffness (k), damping (d) and inertia (m)

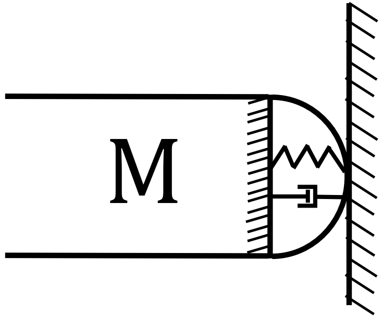
$$f(t) = k \cdot \boldsymbol{x}(t) + d \cdot \dot{\boldsymbol{x}}(t) + m \cdot \ddot{\boldsymbol{x}}(t)$$

Laplace Transform

$$F(s) = \underbrace{(k + d \cdot s + m \cdot s^2)}_{\text{Impedance of the spring-damper-mass system}} \cdot X(s)$$

Impedance of the spring-damper-mass system



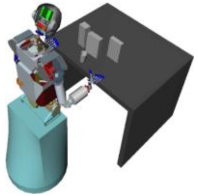


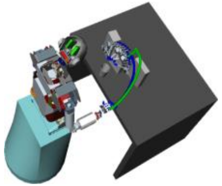
Control of ARMAR-Robots

- Joint space control
- Cartesian space control
- Hybrid position/force control
- Impedance control: Open the fridge/dishwasher
- Image-based control (visual servoing)
- Image and force-based control
- Haptic-based control (haptic servoing)

Execution of Manipulation Tasks

Grasping Task → Scene Representation → Motion Planning → Sensor-based Execution





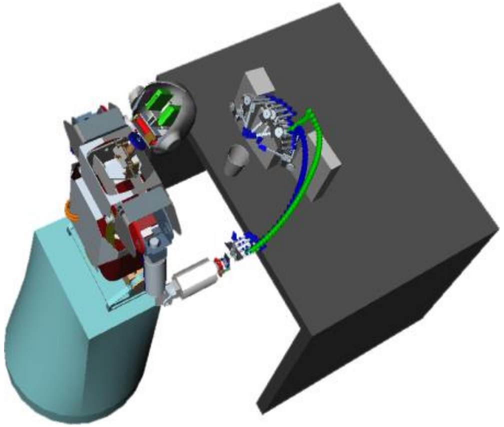
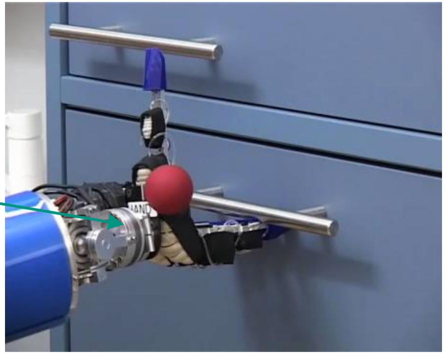


Image-based Position Control for Grasping

Sensors

- Force/torque sensor on both wrists
- Stereo camera system

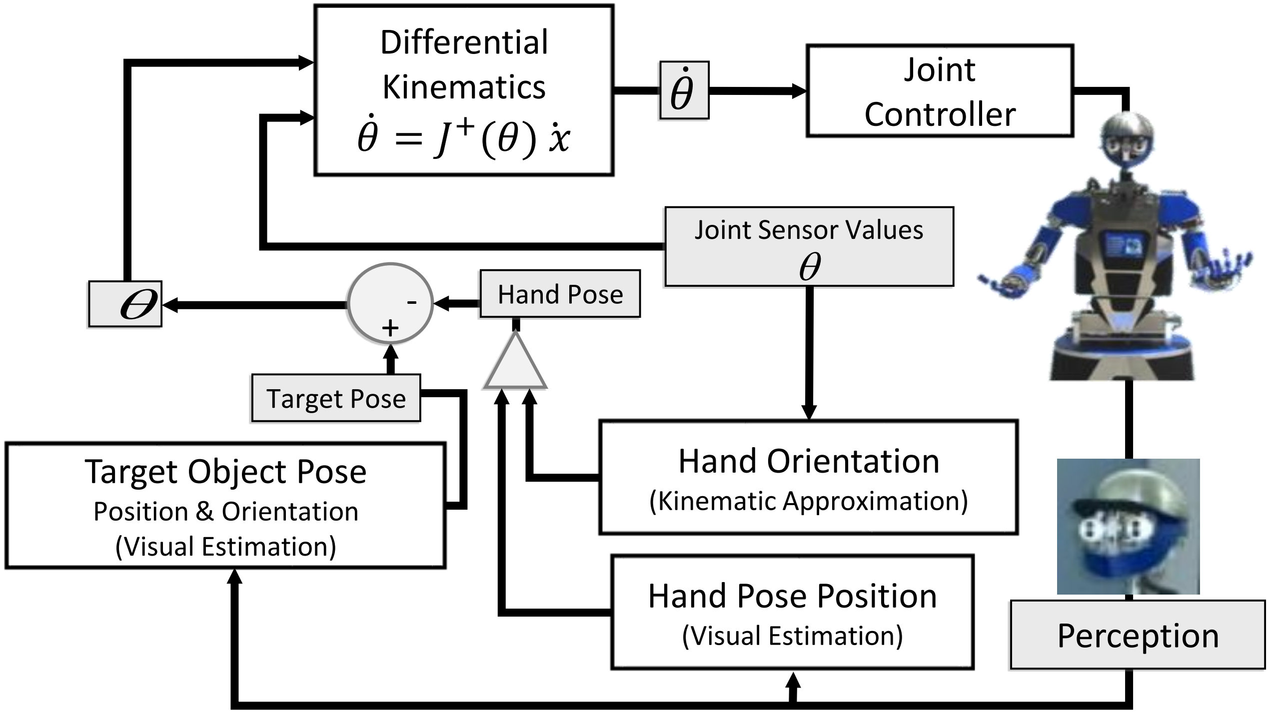


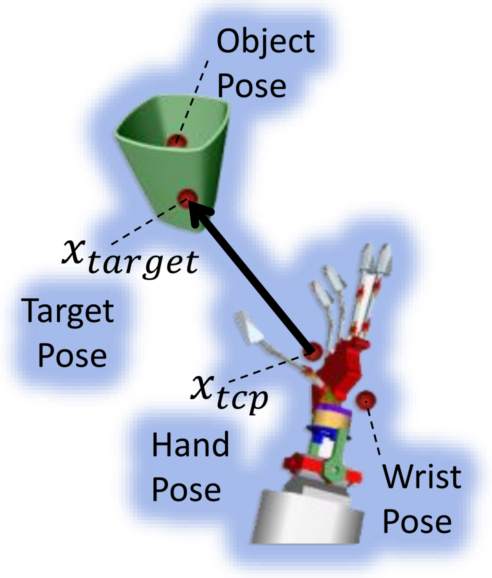


- Tactile skin (upper and lower arm, shoulder)

- Internal sensors (joint angle sensors)

Position-based visual servoing





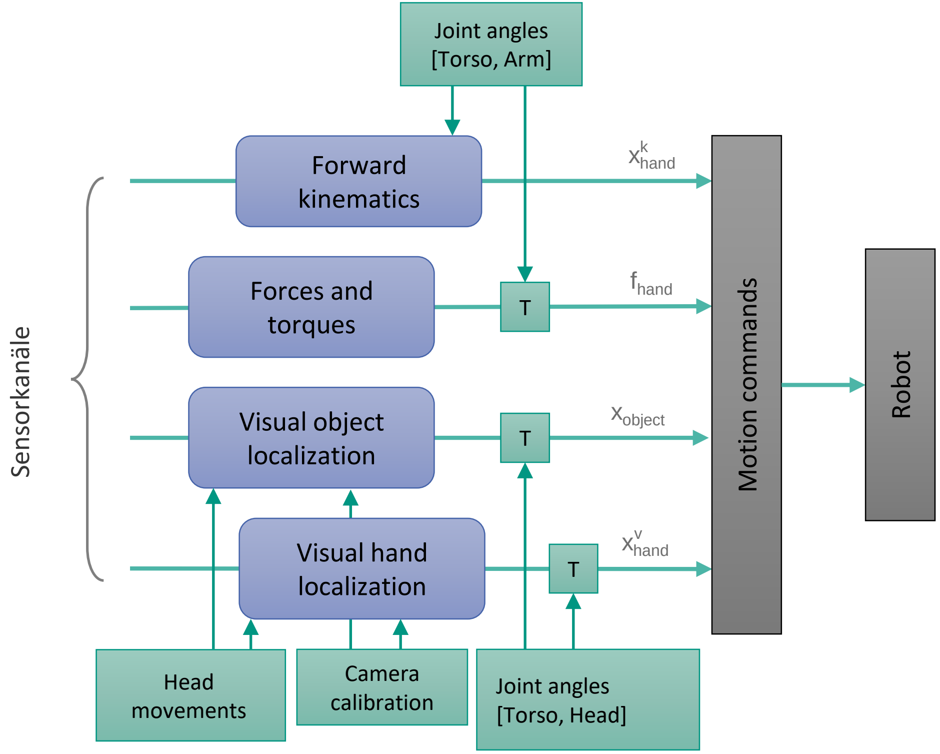
$$\dot{\theta} = J^+(\theta)\dot{x}$$

$$\delta^t = x^t_{vision} - x^t_{kinematic}$$

$$x^{t+1}_{tcp} = x^{t+1}_{kinematic} + \delta^t_{tcp}$$

Sensor-based Execution of Manipulation Tasks

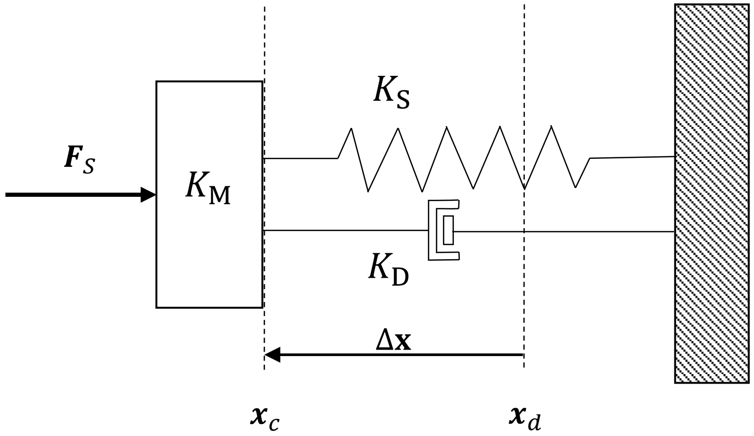
- Image-based execution
  - Model knowledge
- Sensors
  - Force/Contact
  - Cameras
  - Internal sensors

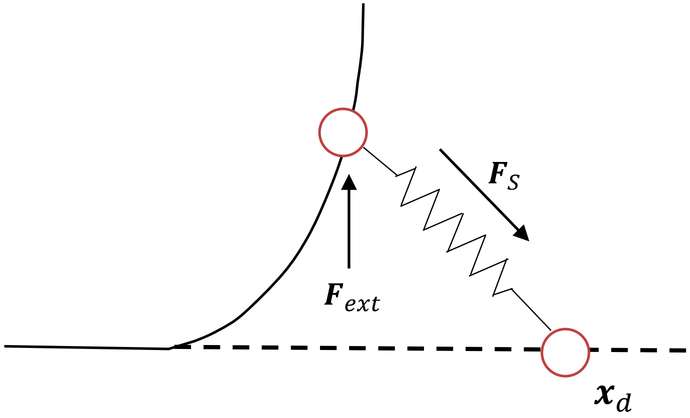


Force Control

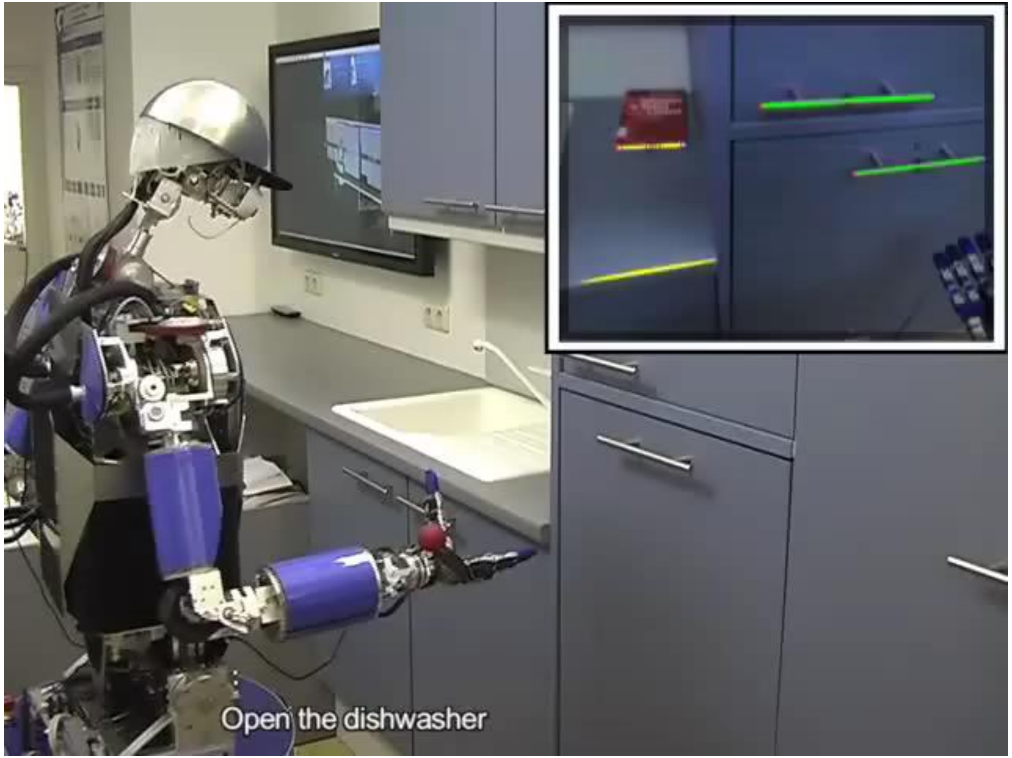
Impedance control

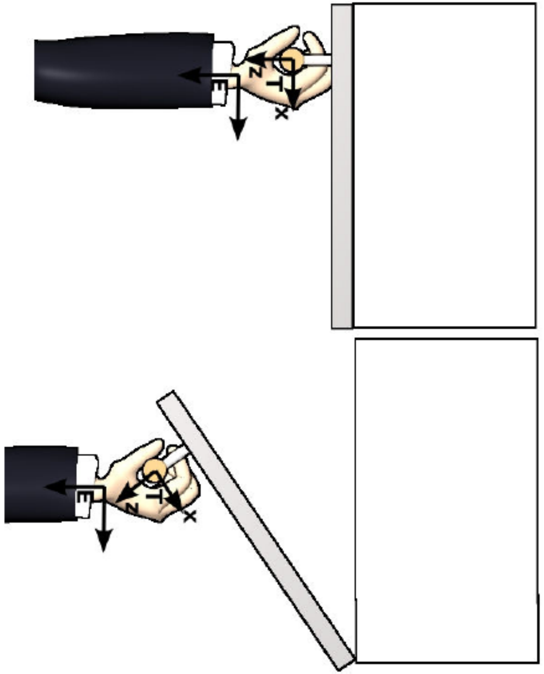
- Control of the relationship between applied force and change in position (i.e. speed) on contact with the environment!
- Speed-based simplifications: Stiffness & damping control



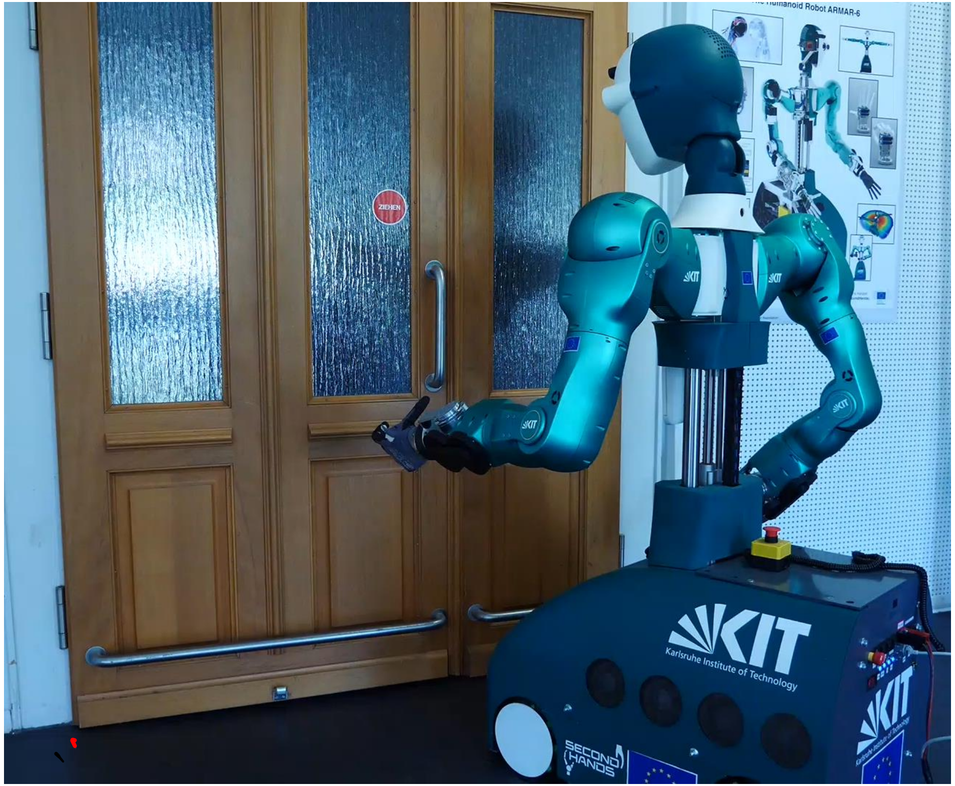


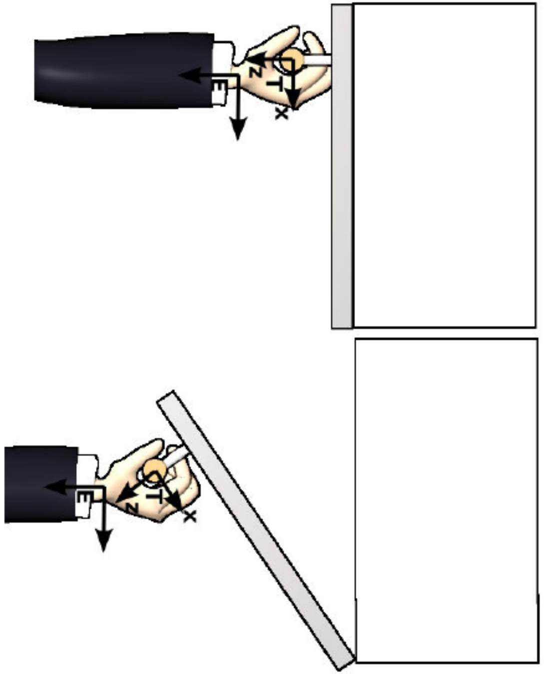
Impedance Control (Open Door)





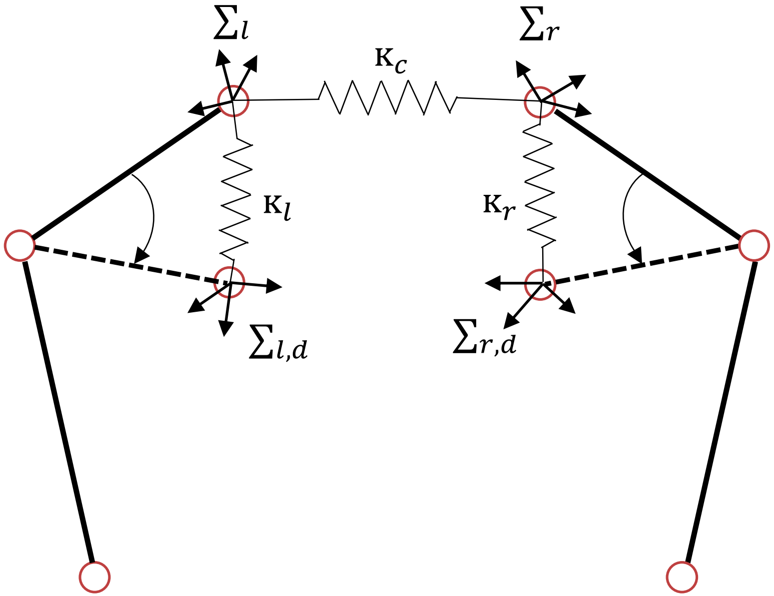
Impedance Control (Open Door)





Bimanual Impedance Control

- Additional coupling stiffness between the end effectors
- Stiffnesses must be compatible

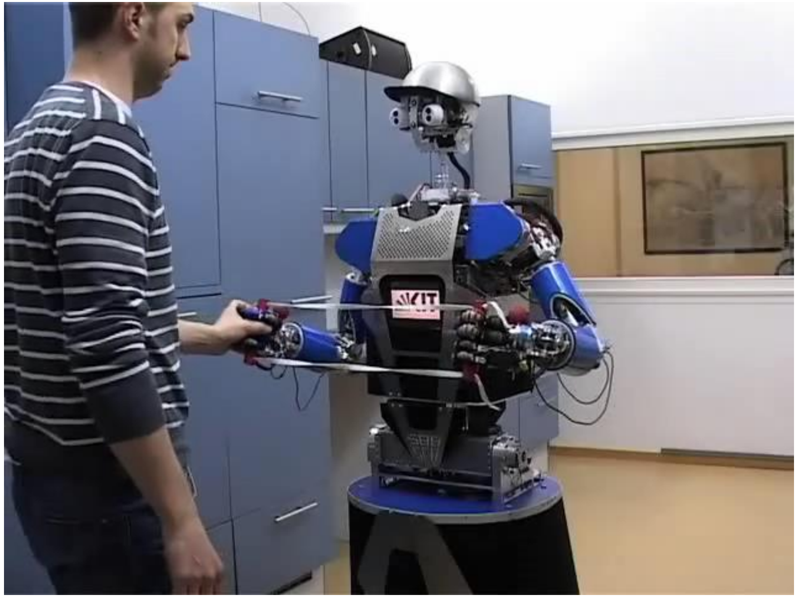


Bimanual Manipulation

- Decoupled manipulation
  - No direct coupling of the arms
  - Independent trajectories

- Coupled manipulation
  - Leader-Leader: Mutual path change
  - Leader-Follower: Path of the follower arm changes when the leader arm is deflected

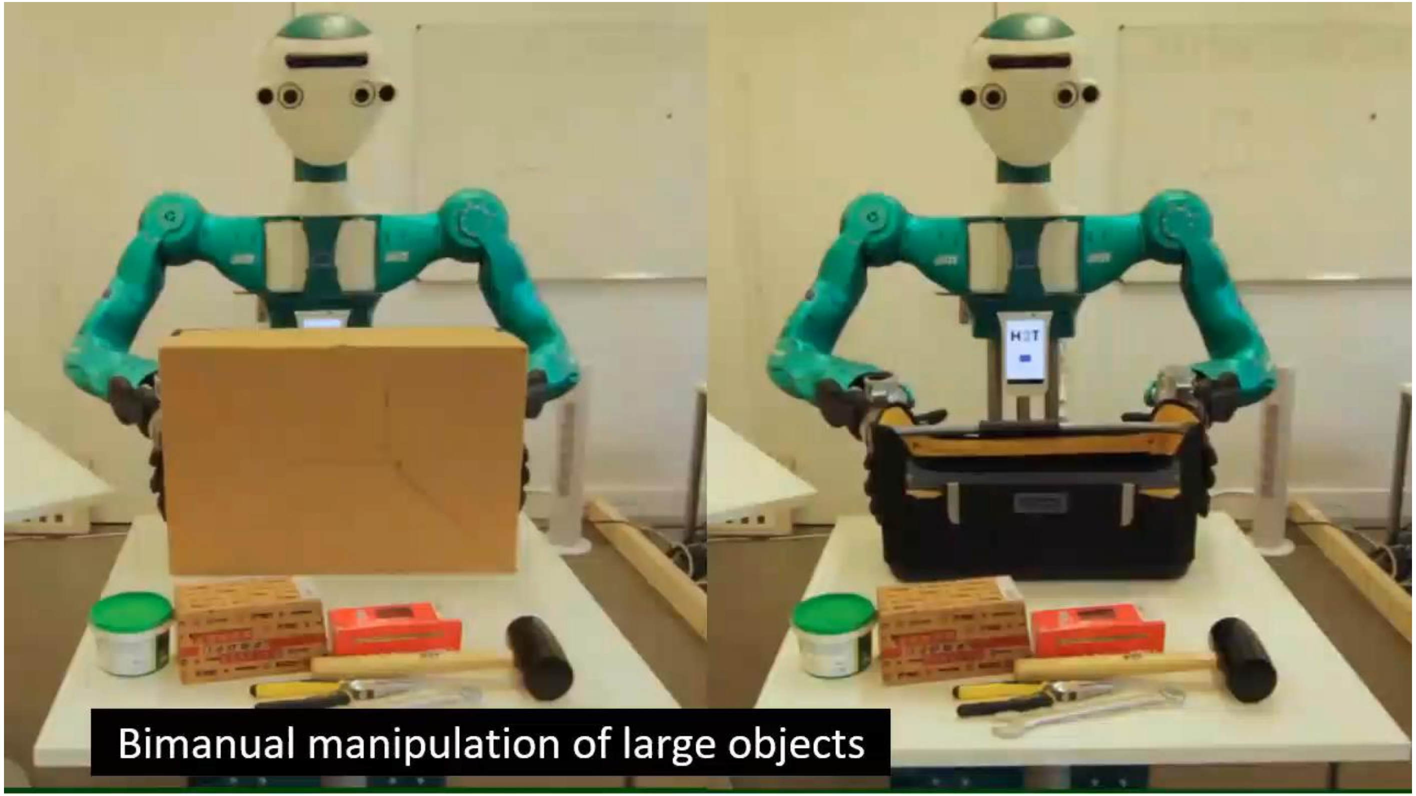
Compliant and Rigid Coupled Manipulation



Compliant Coupled Manipulation

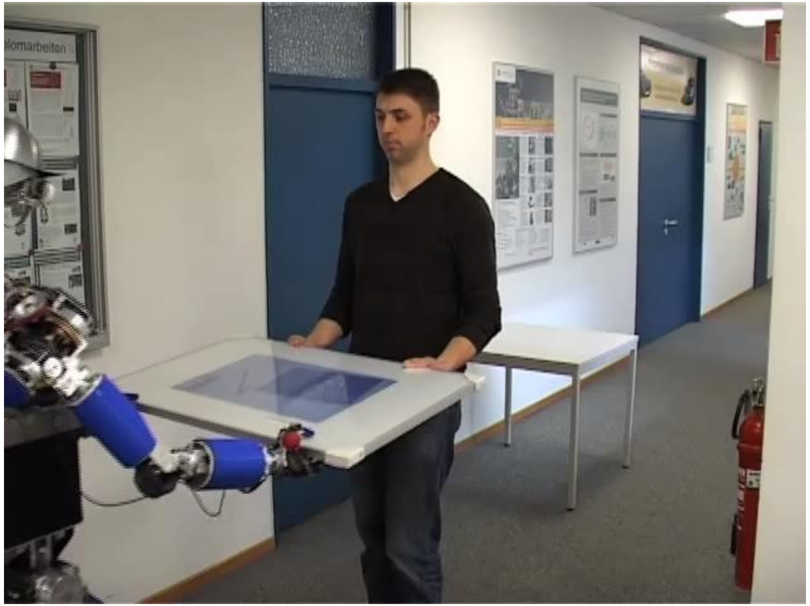
Rigidly Coupled Manipulation

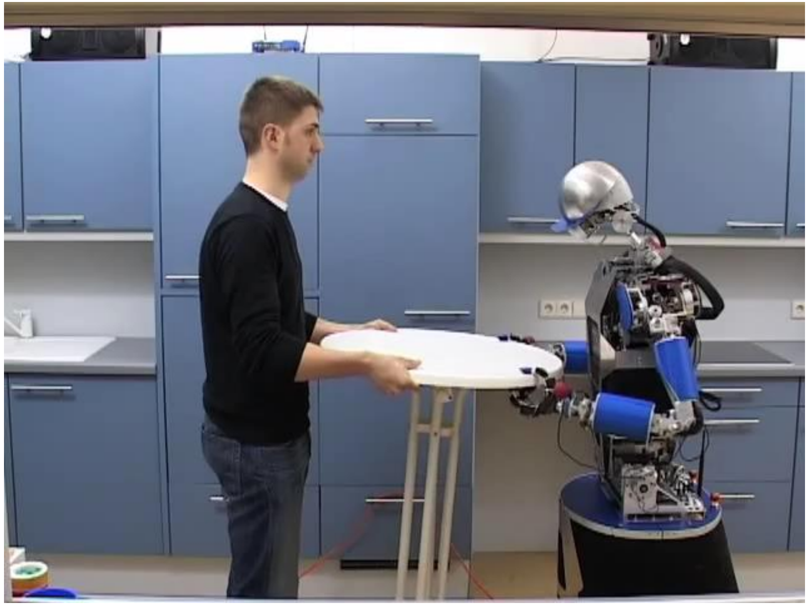
Rigidly Coupled Manipulation



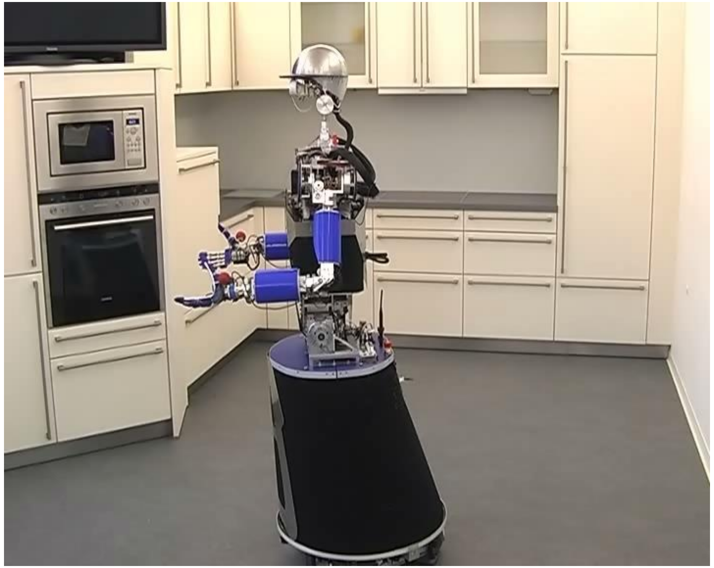
Human-Robot Colaboration

- Force/position control

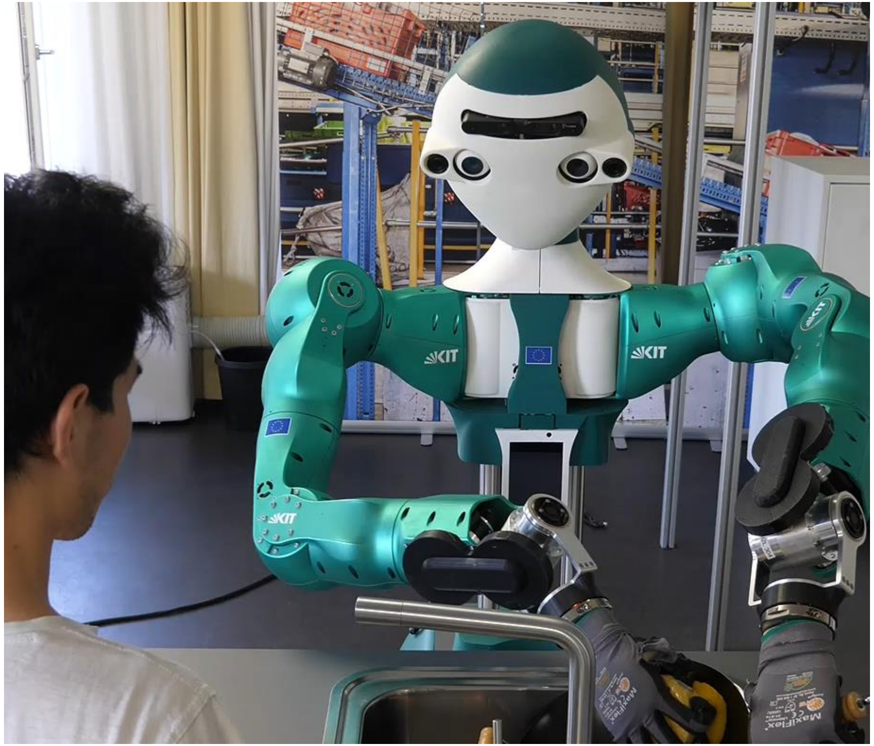




Human-Robot Colaboration



Physical Human-Robot Interaction



German Terminology

English	German
Controller	Regler
Control input	Stellgröße
System output	Ausgangsgröße
Disturbance	Störgröße
Reference	Führungsgröße
Feedback	Rückführgröße
Control error	Regeldifferenz
Closed loop control	Regelung mit geschlossener Schleife
Open loop control	Regelung mit offener Schleife (Steuerung)
Plant	Strecke
Laplace transform	Laplace-Transform
Torque control	Drehmomentregelung

Bibliography

[1] Otto Föllinger, „Regelungstechnik: Einführung in die Methoden und ihre Anwendung“, ISBN: 9783778529706

[2] Lynch, Kevin M., and Frank C. Park. *Modern Robotics*. Cambridge University Press, 2017